Home Made Four-Point Probe: Case Studies of the Wobbly A and B Probes

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ABSTRACT
A simulation on the effect of probe deviation on sheet resistivity value \( R_s \) of Cu/Ni thin film was carried out in a home-made four-point probe (HM-FPP) type. This began by solving the \( R_s \) formula for normal probes, and then for wobbly probe when it was either A, or both A and B. The formula was implemented on a thin layer of Cu/Ni, which was a low temperature sensor material obtained from electrodeposition for 60s assisted by a 200G magnetic field at a current density of 0.07A/mm\(^2\). An electric current of 0.20118A was flown from probe A to D in order to produce a potential difference between probe C and D of 0.0005 volts. Furthermore, the distance between the probes was 5 mm and the deviation of each probe A and B were simulated from -0.5 mm to 0.5 mm. The maximum allowable limit for the relative error of \( R_s \) or \( S_{R_s} \) is 5%. The results showed that the ideal \( R_s \) value was 0.113 ohm/sq. Furthermore, for HM-FPP in which the wobbly probe only A, there is no problem encountered with the variation of the deviation because all \( S_{R_s} \) are less than 5%. For wobbly probes A and B, if they are on the same side of the center point of each probe, the maximum allowable deviation is 0.3 mm. The \( S_{R_s} \) for this case were 4.6%. However, if they are on different sides of the center point of each probe, the maximum allowable deviation is 0.1 mm with \( S_{R_s} \) of 2.9%. With these results, HM-FPP craftsmen must be more careful in making the size of the probe hole.

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I. Introduction

Currently, FPP (Four Point Probe) has been an interdisciplinary sheet resistivity characterization tool in materials science, semiconductor industry, geology, physics, and others. Furthermore, it is used for research in both fundamental types and applications. For example, FPP is widely used to characterize semiconductor thin film materials, to describe the inner condition of the material by measuring the resistivity from the surface [1]. It also plays a role in explaining changes in chemical bonds in materials because the resistivity is inversely proportional to the density and the mobility of charge carriers [2].

Several companies have produced this tool such as Jandel [3], Everbeing [4], Polytec [5], Semilab [6], and others. These tools can be made individually, and are commonly known as home made four-point probes (HM-FPP) [7]–[9]. There are several things that need to be considered when designing a 4-point probe including the probe material, the surface area of the probe tip, the force of the spring that presses the probe, and the area of the probe hole.

The choice of probe material is related to the consideration of the ease of transport of electric current and proportionality between the voltage and current [10]. Generally the material used was copper metal, however, others such gold, silver, nickel and their combination are also used [11]–[13]. However, one of the disadvantages of copper is that corrosion can occur on the surface and when this happens it reduce its conductivity. One of the recommended ways is to clean the surface of the probe from corrosion. In addition, the surface of the probe can
also be coated with other similar or dissimilar conductive materials using the electroplating method.

A probe tip surface that is too wide causes a lot of current to be retained between the interface and the substrate surface. As a result, the current cannot flow smoothly and heat occurs at the meeting point between these two materials thereby providing additional resistance. Furthermore, when the surface of the probe tip is too narrow, it becomes too tapered in order to injure the sample surface when attached to the substrate.

A good spring is one that has sufficient spring constant to press the probe against the sample surface. Furthermore, a spring with a large spring constant would press too hard on the sample surface, thereby damaging it. However, when the spring constant is too small, it becomes too weak to press against the sample surface thereby allowing electric current to flow less smoothly from the probe to the sample.

A good size of the probe hole which enables it to moves back and forth is a bit larger than its size. This causes a deviation of the probe tip from the center point whenever it’s been lowered against the sample surface. Deviation can be left or right, therefore when the probes deviate, the distance between them changes, thereby affecting the sheet resistivity of the sample. For manufacturer FPP, usually this error has been minimized in order to ensure the problem only comes from errors that occur from the ratio of sample size and distance between probes, as well as the position of the edge of the probe to the edge of the material [2], [14]–[16].

In this study, a simulation of the effect of probe hole size on the resistivity value of thin film strips was measured using the HM-FPP four-point probe. This began by solving the equation in relation to the voltage between the point of probe B and C and the current flowing through probe A and D in order to obtain the equation for \( R_s \). Furthermore, from the formula, the input distance between probes \( s \) and the deviation of the probe from the center point a of the deviated probe only probe A for the first case, and then probe A and probe B for the second case.

The formula was implemented to determine the resistivity of Cu/Ni thin film, which is a low temperature sensor material obtained from electrodeposition for 60s assisted by a 200G magnetic field at a current density of 0.07A/mm². Furthermore, 5 mm of \( s \) and a from -0.5 to 0.5 mm were selected. From the results, the tolerable limit of probe deviation a will be determined at a confidence level above or equal to 95%, or a maximum relative error of 5%. These results also serve as a recommendation for HM-FPP makers to consider the maximum probe hole area that can be used to obtain the correct \( R_s \) value.

II. Theory

When an electric current with a current density \( J \) flows through the conductor, the magnitude of the electric field is proportional to the current density [17]–[19].

\[
\vec{E} = \rho J
\]

By expressing the electric field as a potential gradient, and the electric field pointing in a radial direction, then

\[
\rho J = -\frac{dV}{dr}
\]

The electric potential results derived from the integration of the eq. (2),

\[
V = -\int \rho l \frac{dr}{A}
\]

Figure 1 shows the four-point probe. In this tool there are 4 probes, namely A, B, C and D alongside the distance between the probes \( s \). The two outermost, A and D, are probes where current flows from A to D. Due to point D not being grounded, A is a source of positive current while probe B is a source of negative current. Meanwhile, two probes B and C were used for measuring potential difference.

![Figure 1. The principle of the Four-point probe [20]](image)

During current flow, the potential difference between points B and C is contributed by the positive current source at A and the negative current source at D. For positive current coming from probe A, or \( V_{BC^+} \):

\[
V_{BC^+} = -\rho l \int_{x_{AB}}^{x_{AC}} \frac{dr}{2\pi r t}
\]

\[
= -\frac{\rho l}{2\pi t} [ln x_{AC} - ln x_{AB}] = -\frac{\rho l}{2\pi t} ln \left( \frac{x_{AC}}{x_{AB}} \right)
\]

The potential difference between points B and C and the negative current source at D or:

\[
V_{BC^-} = -\rho (-l) \int_{x_{BD}}^{x_{CD}} \frac{dr}{2\pi r t}
\]

\[
= -\frac{\rho (-l)}{2\pi t} [ln x_{CD} - ln x_{BD}] = -\frac{\rho l}{2\pi t} ln \left( \frac{x_{BD}}{x_{CD}} \right)
\]
The total potential difference between points B and C was determined. For further determination of \( V_{BC} \) only the magnitude was determined to be positive.

\[
V_{BC} = \frac{\rho I}{2\pi r} \ln \left( \frac{x_{AC} - x_{BD}}{x_{AB} - x_{CD}} \right)
\]

Due to the low thickness of \( t \), it was difficult to measure \( \rho / t \). A new quantity called sheet resistivity, \( R_s \), was used. Therefore,

\[
R_s = \frac{2\pi}{\ln \left( \frac{x_{AC} - x_{BD}}{x_{AB} - x_{CD}} \right)} V_{BC} t
\]

The relative error for \( R_s \) is obtained by comparing the \( R_s \) value under ideal conditions where the probes do not deviate.

\[
S_R = \left| \frac{R_s - R_{si}}{R_{si}} \right| \times 100\%
\]

where \( R_{si} = \) ideal sheet resistivity, and \( R_{si} = \) sheet resistivity to-i

III. Method

Experiments were carried out according to the following procedure. First, input current \( I = 0.20118 \) A and voltage \( V = 0.005 \) volt was determined using eq. (7). Afterwards, the distance between probes, \( s = 5.0 \) mm and \( a \) were run from \(-0.5\) to \(0.5 \) mm in \(0.1 \) mm increments. For ideal conditions, \( x_{AB} = x; \ x_{AC} = 2s; \ x_{CD} = s; \ x_{BD} = 2x \). For case 1, where the condition of probe A was wobbly and the others were steady, then \( x_{AB} = s \pm a; \ x_{AC} = 2x \pm a; \ x_{CD} = s; \ x_{BD} = 2x \pm a \). The research was continued in case 2, where probes A and B were wobbly and the others were steady, \( x_{AB} = s \pm 2a; \ x_{AC} = 2x \pm a; \ x_{CD} = s; \ x_{BD} = 2x \pm a \). The condition is showed in Figure 2. Furthermore, the value of \( R_s \) and \( S_R \) were determined according to eq. (7) as well as eq. (8). The \( R_s \) data obtained from the two conditions were then analyzed to determine the maximum allowable value of \( a \), to produce \( R_s \) with a relative error of 5%.

IV. Results and Discussion

Sheet resistivity formulation for ideal conditions.

Under ideal conditions, the probe and hole sizes are very precise, as shown in Figure 2.

Therefore, the surface area of the probe is the same as the hole area. By referring to eq. (6) and taking \( x_{AB}, x_{AC}, x_{BC}, \) and \( x_{BD} \) as mentioned above, we get

\[
V_{BC} = \frac{\rho I}{2\pi r} \ln \left( \frac{2s}{s} \right) = \frac{\rho I}{2\pi r} \ln 2 = \frac{\rho I}{\pi r} \ln 2
\]

By substituting \( I = 0.20118 \) A and the voltage \( V = 0.005 \) volt in eq. (9) then \( R_s = 0.0013 \) /sq.

The condition is showed in Figure 2. Furthermore, the value of \( R_s \) and \( S_R \) were determined according to eq. (7) as well as eq. (8). The \( R_s \) data obtained from the two conditions were then analyzed to determine the maximum allowable value of \( a \), to produce \( R_s \) with a relative error of 5%.

The sheet resistivity formulation for probe A is wobbly and the other probes are steady.

This is not an ideal condition. This occurs when the size of the hole of probe A is too large than the probe size, which causes the probe to become wobbly. Schematically, this condition is shown in Figure 3.

\[
V_{BC} = \frac{\rho I}{2\pi r} \ln \left( \frac{2s}{s} \right) = \frac{\rho I}{2\pi r} \ln 2 = \frac{\rho I}{\pi r} \ln 2
\]

Figure 2. Ideal homemade four-point probe. (a) Bottom view, (b) all probes steady

Figure 3. Non ideal homemade four-point probe. (a) bottom view, (b) Probe A is loose while the others are steady

The shaking of probe A will cause the probe tip to deviate from its normal position therefore the distance between AB and AC changes. Depending on the direction of the tip deviation of probe A, there are 2 alternatives as shown in Table 1.
Probe A moves from left to right.

When probe A starts to move from the position of the left end to the right end, the distance from probe A to B gradually changes from \(s+a\) to \(s-a\), or briefly expressed by \(s \pm a\). The probe distance A to C also changes from \(2s+a\) to \(2s-a\) or \(2s \pm a\). Meanwhile, the distance from probe B to D remains \(2s\) and probe C to D remains \(s\). Then substituting that value into Eq. (7), this derives

\[
V_{BC} = \frac{\rho l}{2\pi} \ln \left( \frac{x_{AC} \mp a}{x_{AB} \mp a} \times \frac{x_{BD}}{x_{CD}} \right)
\]

(10)

\[
= \frac{\rho l}{2\pi} \ln \left( \frac{2s \mp a}{s \pm a} \times \frac{2s}{s} \right)
\]

(11)

\[
R_s = \frac{2\pi}{\ln \left( \frac{2s+a}{s \pm a} \times \frac{2s}{s} \right)} \frac{V_{BC}}{I}
\]

Probe A moves from right to left.

If probe A starts to move from the right end to the left end, then the distance from probe A to B gradually changes from \(s-a\) to \(s+a\). The probe distance A to C also changes from \(2s-a\) to \(2s+a\) or. Substituting that value into Eq. (7) this derives

\[
V_{BC} = \frac{\rho l}{2\pi} \ln \left( \frac{x_{AC} \mp a}{x_{AB} \mp a} \times \frac{x_{BD}}{x_{CD}} \right)
\]

(12)

\[
= \frac{\rho l}{2\pi} \ln \left( \frac{2s \mp a}{s \pm a} \times \frac{2s}{s} \right)
\]

(13)

The description of the distribution of \(R_s\) from eq. (12) and (13) are shown in Table 2. From Table 2 it was observed that when probe A is loose, the value of \(R_s\) will vary. For probe A moving to the right, the \(R_s\) value increases from 0.108 ohm/sq (when the probe is 0.5 mm to the left of the center point) to 0.117 ohm/sq (when the probe is 0.5 mm to the right of the center point), and vice versa. When probe A moves to the left then the value of \(R_s\) decreases with the initial and final values, similar to probe A which moves from left to right. Furthermore, the value of \(R_s\) ranges from 0.108 ohm/sq to 0.117 ohm/sq while the ideal value is 0.113 ohm/sq. The largest relative error for \(R_s\) for either right or left motion probe A was 3.8%. With this error, the loosening of the hole of probe A to enhance its freedom to move to the left of 0.5 mm and to the right of 0.5 mm from the center of the probe can still be tolerated.

Table 2. The distribution of \(R_s\) values for loosely attached probe A

<table>
<thead>
<tr>
<th>Distance (a) (mm)</th>
<th>Ideal conditions</th>
<th>Probe A moves from left to right</th>
<th>Probe A moves from right to left</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(R_s) (Ω/sq)</td>
<td>(R_s) (Ω/sq)</td>
<td>(S_{R_s}) (%)</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.113</td>
<td>0.108</td>
<td>3.8</td>
</tr>
<tr>
<td>-0.4</td>
<td>0.113</td>
<td>0.109</td>
<td>3.0</td>
</tr>
<tr>
<td>-0.3</td>
<td>0.113</td>
<td>0.110</td>
<td>2.2</td>
</tr>
<tr>
<td>-0.2</td>
<td>0.113</td>
<td>0.111</td>
<td>1.5</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.113</td>
<td>0.112</td>
<td>0.7</td>
</tr>
<tr>
<td>0.0</td>
<td>0.113</td>
<td>0.113</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.113</td>
<td>0.113</td>
<td>0.7</td>
</tr>
<tr>
<td>0.2</td>
<td>0.113</td>
<td>0.114</td>
<td>1.4</td>
</tr>
<tr>
<td>0.3</td>
<td>0.113</td>
<td>0.115</td>
<td>2.1</td>
</tr>
<tr>
<td>0.4</td>
<td>0.113</td>
<td>0.116</td>
<td>2.8</td>
</tr>
<tr>
<td>0.5</td>
<td>0.113</td>
<td>0.117</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Probes A and B are loose and other probes are steady

As mentioned in part 2, this is also not an ideal condition. This occurs when the size of the hole A and B are too large than the size of probes A and B, which causes the probe to become wobbly. Schematically, this condition is shown in Figure 4.
The motion pattern of probes A and B

Probes A and B move from left to right.

When probes A and B move from left to right, the initial distance of probe A to B is s while A to C is \((2s + a)\). When probe A reaches the right edge of the hole, the distance of probe A to B remains s while the distance from A to C is \((2s - a)\). Meanwhile, for probe B when its position is on the left edge of the hole, the distance from B to D is \((2s + a)\), and when probe B is on the right edge of the hole, the distance from B to D is \((2s - a)\). The distance between probe C and probe D remains s. Therefore, according to eq. (6)

\[
V_{BC} = \frac{\rho l}{2\pi} \ln \left( \frac{x_{AC} \pm a}{x_{AB} \times x_{CD}} \right)
\]

\[(14)\]

\[
= \frac{\rho l}{2\pi} \ln \left( \frac{2s \pm a}{s} \times \frac{2s \pm a}{s} \right)
\]

while the value of \(R_s\) becomes

\[
R_s = \frac{2\pi}{\ln \left( \frac{2s + a}{s} \times \frac{2s + a}{s} \right)} \frac{V_{BC}}{I}
\]

(15)

Probe A moves from left to right and B from right to left.

When the position of probe A is on the edge of the left hole and moves to the right, and the position of probe B on the edge of the right hole moves to the left, the distance from probe A to B is initially \(s + 2a\) and finally \(s - 2a\). Meanwhile, the distance between probe B to D is initially \(2s - a\) and finally \(2s + a\), and the probe distance C to D is initially \(s\) and finally remains \(s\). According to eq. (6), the potential difference between points B and C is

\[
V_{BC} = \frac{\rho l}{2\pi} \ln \left( \frac{x_{AC} \pm a}{x_{AB} \pm 2a} \times \frac{x_{BD} \pm a}{x_{CD}} \right)
\]

\[(16)\]

\[
= \frac{\rho l}{2\pi} \ln \left( \frac{2s \pm a}{s \pm 2a} \times \frac{2s \pm a}{s} \right)
\]

while the value of \(R_s\) becomes

\[
R_s = \frac{2\pi}{\ln \left( \frac{2s + a}{s} \times \frac{2s + a}{s} \right)} \frac{V_{BC}}{I}
\]

(19)

Probe A moves from right to left and probe B from left to right.

When the position of probe A starts at the edge of the right hole and moves to the left while probe B is initially on the edge of the right hole and moves to the left until it reaches the edge of the left hole, the distance from probe A to B is initially \(s - 2a\) and finally \(s + 2a\). Meanwhile, the distance between probe B to D is initially \(2s + a\) and finally \(2s - a\), and the distance from probe C to D is initially \(s\) and finally remains \(s\). According to eq. (6), the potential difference between points B and C is

\[
V_{BC} = \frac{\rho l}{2\pi} \ln \left( \frac{x_{AC} \pm a}{x_{AB} \pm a} \times \frac{x_{BD} \pm a}{x_{CD}} \right)
\]

\[(20)\]

\[
= \frac{\rho l}{2\pi} \ln \left( \frac{2s \pm a}{s \pm a} \times \frac{2s \pm a}{s} \right)
\]
while the value of $R_s$ becomes

$$R_s = \frac{2\pi}{\ln\left(\frac{2s + a}{s} + \frac{2s + a}{s} + a\right)} \frac{V_{es}}{I}$$

(21)

A description of the simulation results from equations (15), (17), (19), and (21) for the values of $s=5$ mm and $a=0.5$ mm as shown in Table 4.

<table>
<thead>
<tr>
<th>Distance $a$ (mm)</th>
<th>Ideal conditions</th>
<th>Probe A from left to right</th>
<th>Probe A from left to right and probe B from right to left</th>
<th>Probe A and B right to left</th>
<th>Probe A from right to left and probe B from left to right</th>
<th>$S_{R_s}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.113</td>
<td>0.105</td>
<td>0.130</td>
<td>0.122</td>
<td>8.0</td>
<td>7.6</td>
</tr>
<tr>
<td>0.4</td>
<td>0.113</td>
<td>0.107</td>
<td>0.126</td>
<td>0.109</td>
<td>6.3</td>
<td>6.0</td>
</tr>
<tr>
<td>0.3</td>
<td>0.113</td>
<td>0.108</td>
<td>0.123</td>
<td>0.115</td>
<td>4.6</td>
<td>4.5</td>
</tr>
<tr>
<td>0.2</td>
<td>0.113</td>
<td>0.110</td>
<td>0.119</td>
<td>0.113</td>
<td>3.0</td>
<td>2.9</td>
</tr>
<tr>
<td>0.1</td>
<td>0.113</td>
<td>0.111</td>
<td>0.116</td>
<td>0.115</td>
<td>1.5</td>
<td>1.2</td>
</tr>
<tr>
<td>0.0</td>
<td>0.113</td>
<td>0.113</td>
<td>0.113</td>
<td>0.113</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.2</td>
<td>0.113</td>
<td>0.116</td>
<td>0.106</td>
<td>0.114</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>-0.3</td>
<td>0.113</td>
<td>0.118</td>
<td>0.103</td>
<td>0.108</td>
<td>4.1</td>
<td>4.0</td>
</tr>
<tr>
<td>-0.4</td>
<td>0.113</td>
<td>0.120</td>
<td>0.100</td>
<td>0.107</td>
<td>5.4</td>
<td>5.6</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.113</td>
<td>0.122</td>
<td>0.097</td>
<td>0.105</td>
<td>6.6</td>
<td>6.9</td>
</tr>
</tbody>
</table>

For probes A and B which both move from left to right or from right to left a deviation limit of ± 0.3 mm is allowed. With this limit, $R_s = 0.108 \text{ /sq}$ with a relative error of 4.1% when both probes deviate to the left from the center point and $R_s = 0.118 \text{ /sq}$ with a relative error of 4.6%, when both probes deviate to the right from the center point.

Furthermore, probe A which moves from left to right was paired with probe B which moves from right to left, allowing the deviation of probe A to be only ± 0.1 mm. When probe A 0.1 mm to the left of the center point and probe B 0.1 mm to the right of the center point, the value of $R_s = 0.116 \text{ ohm/sq}$ was obtained with a relative error of 2.9%. Alternatively, when probe A is 0.1 mm to the right of the center point and probe B is 0.1 mm to the left, $R_s = 0.109 \text{ ohm/sq}$ with a relative error of 2.9%. When the deviation of probes A and B is increased to 0.2 mm, a relative error of 5.9% was obtained (that is, if probe A is to the left of the center point and probe B is to the right). Similarly, for probe A which is to the left of the center of the probe and B to the right of the center, a relative error of 5.6% is obtained. This error exceeded the set relative error limit of 5%. Therefore, a probe deviation of 0.2 mm should not occur.

For probe A moving from right to left and B from left to right, the maximum allowable deviation was ±0.3 mm. When probe A was 0.3 mm to the right of the center point and B was 0.3 mm to the left, $R_s = 0.118 \text{ ohm/sq}$ was obtained with a relative error of 4.5%. Meanwhile for probe A which was 0.3 mm to the left of the center point and B 0.3 mm to the right, $R_s = 0.108 \text{ ohm/sq}$ with a relative error of 4.2%.

From the results of the research that has been carried out, it is suggested for craftsmen of home made four point probes to really pay attention to the size of the probe hole. By setting the allowable error limit for $R_s$, 5% to ideal $R_s = 0.113 \text{ /sq}$, the following conditions are obtained. If only probe A is wobbling, then at a maximum deviation of probe A of 5 mm, a maximum relative error of 3.8% is obtained. This is not a problem because the relative error of $R_s$ is less than 5%. For the wobbly probes A and B, the maximum allowable deviation of probes A and B is limited to 0.3 mm. In this case, a relative error of $R_s$ at 4.6% is obtained.

V. Conclusion

This study showed that in the use of a four-point home made probe, the size of the hole must be considered because it affects the sheet resistivity value. By applying a maximum relative error limit of $R_s$ of 5%, the probe deviation of 0.3 mm from probe center point can still be tolerated for probe A which is paired with probe B where both are on the same side of the center point. Furthermore, for probe A which is to the left of the center point paired with B which is to the right of the probe or vice versa, the maximum limit of probe deviation is 0.1 mm. With these
results, HM-FPP craftsmen must be more careful in making the size of the probe hole.

References

Declarations

Author contribution: Dr. Moh. Toifur, M.Si, contributed to developing research ideas and concepts, drafting the manuscript, and critical revision. Dr. Moh. Irma Sukarelawan, M.Pd, contributed to helping prepare the manuscript and the critical revision of the manuscript. Okimis, contributed to developing research ideas and concepts, drafting the manuscript, and critical revision. Dr. Moh. Toifur, M.Si, contributed to developing research ideas and concepts, drafting the manuscript, and critical revision.

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