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A heterogeneous fleet electric vehicle routing model with soft time windows

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ABSTRACT

The emergence of electric vehicles in distribution and logistics activities has brought significant benefits due to their unique characteristics, such as energy-efficient and lower carbon emissions. In the perspective of vehicle routing problem, electric vehicles pose challenging constraints regarding the limited battery capacity, and thus their traveling ranges, and the availability of charging stations. In this paper, we propose a model of the fleet electric vehicle routing problem (EVRP) with soft time windows, where a mixed integer linear programming framework is implemented in model formulation. The objective of mathematical programming is to minimize the total operational cost, which consists of a fixed cost, a traveling cost, a battery charging cost, and probably a penalty cost due to time window violation. We implement our model in two simple cases, namely homogeneous and heterogeneous fleets EVRPs, characterized by loading and battery capacities. Each case consists of one depot, five customers, two electric vehicles, and two charging stations. Optimal routes are obtained using the well-known branch-and-bound method under Lingo 17.0. It is found that the existence of charging stations may affect the routes selection and the implementation of soft time windows rather than hard time windows has been proven to increase the feasibility of routing problem.

1. Introduction

The confluence of technological progress, environmental concerns, supportive government policies, infrastructure development, and changing consumer preferences has driven the emergence and rapid growth of electric vehicles in recent years [1, 2, 3]. As reported in [4], the emergence of electric vehicles (EVs) has also brought new perspectives and considerations to the study of Vehicle Routing Problems (VRPs). As the transportation sector shifts towards cleaner and more sustainable alternatives, integrating EVs into VRPs has gained significant attention. From a modeling point of view, solving electric VRPs (EVRPs) efficiently is a challenging task due to their combinatorial nature. Modeling routing problems with EVs involves addressing range limitations, charging infrastructure, energy pricing, battery degradation, uncertainties, and the integration of renewable energy sources [5, 6, 7]. Developing effective models and algorithms that consider these challenges is crucial for optimizing the routes, minimizing energy consumption, and ensuring the efficient operation of EV fleets.

The state of the art in modeling EVRPs involves considering various factors such as battery state of charge [8], time windows, charging infrastructure [9], and energy consumption [10]. Advanced models incorporate optimization techniques to minimize costs, optimize routes, and maximize charging station utilization. Additionally, recent research explores the integration of renewable energy sources and the development of intelligent algorithms to enhance the efficiency and sustainability of EVRP solutions [11]. Cataldo-Díaz et al. [12] addresses the EVRP with battery state of charge, where it involves delivering to customers with specific demands and time windows. The objective is to minimize cost (time) by maximizing charging station visits, avoiding battery degradation. Experimental results demonstrate the efficiency of these formulations in various instances, considering battery degradation. Zuo et al. [13] proposes a new mixed-integer linear programming (MILP) model for EVRP with time windows considering concave nonlinear charging function. Experimental study demonstrated that the proposed model provides better logistics schedules for Evs with efficient charging time utilizations. From the application point of view, EVRP models have been implemented in scheduling of a fleet of electric taxis [14] and a fleet of hybrid buses [15].

Vehicle Routing Problems (VRPs) are a class of combinatorial optimization problems that deal with efficiently planning routes for a fleet of vehicles to serve a set of customers while minimizing overall costs or maximizing efficiency. VRPs find applications in various real-world scenarios, such as package delivery, transportation, waste collection, and more. The main objective is to find optimal or near-optimal routes that satisfy specific constraints, such as capacity limits, time windows, and distance limitations. There are several VRP variants, each with specific characteristics and constraints [16, 17]. Some common VRP variants include capacitated VRP (CVRP) where each vehicle has a limited capacity, VRP with time windows (VRPTW) which extends CVRP by adding time windows in which customers must be serviced, multi-depot VRP (MDVRP) if there are multiple depots and each vehicle is assigned to a specific depot, periodic VRP (PVRP) which deals with recurring routing problems, where the same set of customers must be serviced in each period, VRP with pickup and delivery (VRPPD) which involves picking up goods from one location and delivering them to another, often with different time windows and capacity constraints for pickups and deliveries, and split delivery VRP (SDVRP) which allows a customer's demand to be split across multiple vehicles, enabling more flexible deliveries.

VRPs belong to the class of NP-hard problems, which means that finding an optimal solution within a reasonable amount of time is computationally infeasible for large problem instances. Thus, due to their inherent complexity and computational challenges, most VRPs models are solved heuristically [18]. Futalef et al. [19] and Zhang et al. [20], respectively, implemented genetic algorithm to solve an online and two-echelon EVRP models, Bruglieri et al. [21] and Sadeghi-Velni et al. [22] proposed variable neighborhood search algorithms to approach a large-scale routing problem with EVs, and Setak & Karimpour [23] exploited a simulated annealing algorithm to efficiently search the solution space of EVRP with time windows and queueing at charging stations. Elahi & Darestani [24] also utilizes a simulated annealing algorithm to handle a periodic EVRP. A combination of ant colony principle and bee colony algorithm were developed in [25] to hierarchically solve a multi-objective EVRP. A matheuristic approach, namely Random Kernel Search, was proposed in [26] to tackle the large-sized MILP model for time windows EVRP with payload and the vehicle speed consideration. Other heuristic approaches include the use of diversity-enhanced memetic algorithm [27], adaptive large neighborhood search algorithm [28, 29], and multigraph-based adaptive heuristic [30] (Karimpour et al., 2023). For comprehensive accounts on the development of EVRP modeling including prospects for the future research trends, there are several systematic literature surveys such as [31, 32, 33]. The bare-bones mathematical models of EVRP are given in [34, 35]. The models were then extended by incorporating soft time windows [36] under the existence of battery swapping stations which require no charging times. Despite advancements in the field, current approaches to nonlinear optimization often result in solutions that are only locally optimal. This limitation highlights a significant gap in achieving globally optimal solutions, underscoring the need for further research in this area. Therefore, this research proposed a model of the fleet EVRP with soft time windows, where a mixed integer linear programming framework is implemented in model formulation

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The contribution of this paper is two-fold. Firstly, we are developing a heterogenous fleet EVRP model with soft time windows using mixed integer linear programming (MILP) under the existence of battery charging stations where charging times must be considered. Routing problems with soft time windows are particularly useful in scenarios where customers' time preferences are not strict or where there is a need to balance service quality with operational efficiency, ensuring that the routes are planned in a way that satisfies both the customers' expectations and the operational constraints of the vehicle fleet [37]. Secondly, we are examining the effect of charging stations availability on routes selection of homogeneous and heterogeneous fleets. An exact approach, i.e., the branch-and-bound method, is applied to solve the optimization model.

2. Methods

2.1. Problem statement

The heterogeneous fleet electric vehicle routing problem with soft time windows (HFEVRP-STW) arises in the context of optimizing the delivery routes for a fleet of EVs with varying characteristics, namely loading capacity, battery capacity, and battery consumption rate. The primary objective is to develop efficient and cost-effective routes that satisfy customer demands while considering the inherent constraints of EVs and accommodating soft time windows for timely deliveries. Problem components include the followings:

1. The fleet comprises EVs with diverse specifications, namely varying loading capacity, battery capacity, and battery consumption rate.
2. A set of customer locations with specific demands must be serviced by the fleet. Each customer has a soft time window during which deliveries can occur without penalties, but flexibility is allowed within predefined limits.
3. Charging stations are strategically located, and the electric vehicles must plan their routes to include necessary charging stops.
4. The primary objective is to minimize the total cost of the delivery routes. The cost includes fixed cost, travel cost, recharging costs, and potential penalties associated with violating soft time windows.
5. Constraints including but not limited to loading and battery capacities constraints for each electric vehicles, route continuity constraints, and soft time windows constraints.

The schematic diagram of the HFEVRP-STW model is illustrated by Fig. 1.

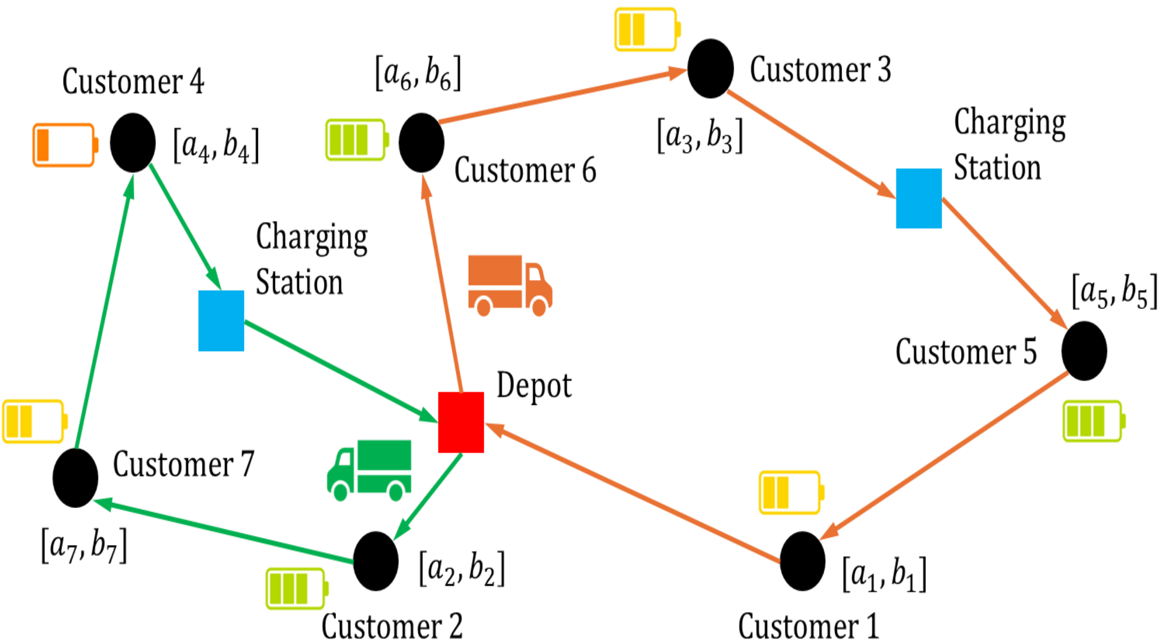

Fig. 1. Schematic diagram the HFEVRP-STW model

2.2. Assumptions and notations

The HFEVRP-STW is formulated as a mixed integer programming problem that considers the objective of minimizing the total cost, subject to the constraints outlined above. To facilitate the formulation and analysis of the HFEVRP-STW model, we impose the following assumptions:

1. Fixed charging capacity: each EV has a predetermined charging capacity, which represents the maximum amount of energy it can store during a charging session.
2. Limited autonomy, where EVs have a limited driving range or autonomy, which means they can only travel a certain distance before requiring recharging.
3. Soft time windows: time windows for customer visits are flexible. However, tardiness incurs penalties, and the optimization algorithm must balance between minimizing penalties and optimizing other cost components.
4. The battery consumption of EVs is measured based only on distance traveled.
5. There are some charging stations available. An EV that has just left a charging station is assumed to have a battery at full capacity. An EV may visit a charging station more than once for battery recharging.
6. Each customer has a specific demand for goods or services and a time window.
7. There is a single depot where all EVs start and end their routes. An EV leaves the depot with a battery at full capacity.

Before stated our model, we introduce some sets, indices, parameters, and variables involved in the model representation.

Sets and indices

$\mathbb{V}$: set of all customers, $\mathbb{V} = \{1,2,\dots,n\}$, $i,j \in \mathbb{V}$
$\mathbb{K}$: set of all EVs, $\mathbb{K} = \{1,2,\dots,m\}$, $k \in \mathbb{K}$
$\mathbb{F}$: set of all charging stations, $\mathbb{F} = \{1,2,\dots,p\}$, $l \in \mathbb{F}$
$\mathbb{U}_1$: set of all customers and charging stations, $\mathbb{U}_1 = \mathbb{V} \cup \mathbb{F}$
$\mathbb{U}_2$: set of all customers, charging stations, and starting depot, $\mathbb{U}_2 = \mathbb{V} \cup \mathbb{F} \cup \{0\}$
$\mathbb{U}_3$: set of all customers, charging stations, and end depot, $\mathbb{U}_3 = \mathbb{V} \cup \mathbb{F} \cup \{n+1\}$
$\mathbb{U}_4$: set of all customers, charging stations, and depots, $\mathbb{U}_4 = \mathbb{V} \cup \mathbb{F} \cup \{0, n+1\}$

Parameters

d_{ij} : distance between nodes i and j (kilometer)
t_{ijk} : travel time between nodes i and j using vehicle k (minute)
v_k : constant velocity of vehicle k (kilometer/minute)
h_k : battery consumption rate by vehicle k (kilowatt per kilometer)
q_i : demand of customer i (kilogram) with $q_i = 0$ for $i \notin \mathbb{V}$
a_i : earliest start of service at customer i (minute)
b_i : latest start of service at customer i (minute)
s_i : service duration at customer i (minute) with $s_0 = s_{n+1} = s_l = 0$
Q_k : loading capacity of vehicle k (kilogram)
$\bar{B}_k$: battery capacity of vehicle k (kilowatt)
g_k : battery charging rate of vehicle k (kilowatt/minute)
c_f : fixed cost for operating vehicle (monetary per unit)
c_t : traveling cost (monetary per kilometer)
c_r : battery recharging cost (monetary per unit)
c_a : penalty cost for earliness (monetary per minute)

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c_b : penalty cost for tardiness (monetary per minute)
ρ : the allowable violation of time windows (minute).

Decision variables

x_{ijk} : binary variable indicating whether vehicle k travels from customer i to j, i.e., $x_{ijk} = 1$ if there is a route from customer i to j and $x_{ijk} = 0$ if otherwise.
τ_{ik} : variable specifying arrival time of vehicle k at customer i (minute)
u_{ik} : variable stating the remaining cargo of vehicle k on arrival at customer i (kilogram)
r_{ik} : variable specifying the remaining battery capacity of vehicle k on arrival at customer i (kilowatt).

In fact, the only decision variable is x_{ijk}, which involves the assignment of vehicles to routes, sequencing of customer visits, and the decision to use charging stations. Other variables are additional but must be determined optimally by the model.

2.3. HFEVRP-STW model

In this part we present the HFEVRP-STW as an extension of models in [34, 35, 36] by allowing more than one electric vehicle (a fleet) to be used and each customer must be visited within a specific soft time window. Similar models for conventional vehicles are given in [38, 39]. The objective of HFEVRP-STW model, as shown in Eq. (1), is to minimize the total cost, which consist of the total fixed cost C_f, the total traveling cost C_t, the total recharging cost C_r, and the total penalty cost C_p (if any). Thus, the total cost function C is given by:

$$\min C \coloneqq C_f + C_t + C_r + C_p, \tag{1}$$

where

$$C_f = \sum_{j \in \mathbb{U}_1} \sum_{k \in \mathbb{K}} c_f x_{0jk}, \tag{2}$$

$$C_t = \sum_{i,j \in \mathbb{U}_1} \sum_{k \in \mathbb{K}} c_t t_{ijk} x_{ijk}, \tag{3}$$

$$C_r = \sum_{l \in \mathbb{F}} \sum_{j \in \mathbb{V}} \sum_{k \in \mathbb{K}} c_r x_{ljk}, \tag{4}$$

$$C_p = \sum_{i \in \mathbb{N}} \sum_{k \in \mathbb{K}} (c_a \max\{0, a_i - \tau_{ik}\} + c_b \max\{0, \tau_{ik} - b_i\}). \tag{5}$$

The first term of the right-hand side of Eq. (5) accounts for penalty cost due to earliness and the second term is for tardiness. Soft time windows offer more advantages than hard ones [39, 40] in terms of flexibility, solution quality, adaptability to variability, and the ability to balance conflicting objectives.

The following constraints are imposed due to standard assumptions asserted by VRP and specific assumptions enforced using EVs.

1. Each customer must be visited exactly once:

$$\sum_{j \in \mathbb{U}_3, j \neq i} \sum_{k \in \mathbb{K}} x_{ijk} = 1, \quad \forall i \in \mathbb{V}, \tag{6}$$

$$\sum_{i\in\mathbb{U}_2, i\neq j}\sum_{k\in\mathbb{K}} x_{ijk} = 1, \quad \forall j \in \mathbb{V}. \tag{7}$$

Eq. (6) ensures that customer i is visited once by any vehicle before travels either to next customer, charging station, or depot. Eq. (7) ascertains that customer j is visited once by any vehicle after travels from previous customer, charging station, or depot.

2. Each vehicle starts and finishes its route at the depot location:

$$\sum_{j\in\mathbb{U}_2} x_{0jk} = 1, \quad \forall k \in \mathbb{K}, \tag{8}$$

$$\sum_{i\in\mathbb{U}_2} x_{i,n+1,k} = 1, \quad \forall k \in \mathbb{K}. \tag{9}$$

Eq. (8) and Eq. (9) account for the requirement that vehicles start their routes from the depot and return to the depot after servicing all customers.

3. Route continuity:

$$\sum_{i\in\mathbb{U}_2, i\neq j}\sum_{k\in\mathbb{K}} x_{ijk} = \sum_{i\in\mathbb{U}_3, i\neq j}\sum_{k\in\mathbb{K}} x_{jik}, \qquad \forall j \in \mathbb{U}_1. \tag{10}$$

Eq. (10) means that the routes followed by vehicles form a connected sequence of customers without any breaks or disjointed segments. The route continuity constraint is an important constraint in VRP models to ensure that vehicles visit customers in a logical and sequential manner. This constraint establishes flow conservation by ensuring that for each customer, the number of incoming visits is equal to the number of outgoing visits.

4. Each vehicle must not visit the same customer:

$$x_{iik} = 0, \qquad \forall i \in \mathbb{V}_4, \forall k \in \mathbb{K}. \tag{11}$$

Eq. (11), however, strengthens the requirement that each customer must be visited exactly once as stated by Eq. (6) and Eq. (7).

5. Consumers' time feasibility and subtour prevention:

$$\tau_{ik} + (t_{ijk} + s_i)x_{ijk} - b_0(1 - x_{ijk}) \leq \tau_{jk}, \qquad \forall i \in \mathbb{U}_2, \forall j \in \mathbb{U}_3, i \neq j, \forall k \in \mathbb{K}, \tag{12}$$

where b_0 is the latest start of service at depot which acts like a big positive number (refer to Eq. (12)).

6. Customer's demand fulfilment:

$$0 \leq u_{jk} \leq u_{ik} - q_i x_{ijk} + Q_k(1 - x_{ijk}), \quad \forall i \in \mathbb{U}_2, \forall j \in \mathbb{U}_3, i \neq j, \forall k \in \mathbb{K}, \tag{13}$$

$$0 \leq u_{0k} \leq Q_k. \tag{14}$$

Eq. (13) and Eq. (14) ensure that all customer demands are met, i.e., the cargo load upon arrival at any node, including the depot, is nonnegative and not exceeding the vehicle's capacity.

7. According to battery state of charge, an EV may visit the charging station:

$$\sum_{j\in\mathbb{U}_3, j\neq i}\sum_{k\in\mathbb{K}} x_{ljk} \leq N_c, \quad \forall l \in \mathbb{F}, \tag{15}$$

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$$\sum_{i\in\mathbb{U}_2, i\neq l}\sum_{k\in\mathbb{K}} x_{ilk} \leq N_c, \quad \forall l \in \mathbb{F}. \tag{16}$$

When the battery state of charge falls below a certain threshold, it may be necessary for the EV to visit a charging station to recharge its battery. Instead of assigning a predefined threshold or the minimum required battery level to complete the remaining route, the model will optimally decide the time of visit to a charging station. Eq. (15) and Eq. (16) allow an EV to visit a charging station at most N_c times, if required.

8. Soft time window means the time window is relaxed, i.e., from $[a_i, b_i]$ to $[a_i - \rho, b_i + \rho]$ for customer i, where $\rho \geq 0$ is the allowable violation of time windows. Thus, we have the following constraint:

$$a_i - \rho \leq \tau_{ik} \leq b_i + \rho, \quad \forall i \in \mathbb{V}, \ \forall k \in \mathbb{K}. \tag{17}$$

An earliness will incur a penalty cost of $c_a(a_i - \tau_{ik})$ and a tardiness will cause a penalty cost of $c_b(\tau_{ik} - b_i)$. The total penalty cost C_p is given in Eq. (5) and must be minimized. Time windows relaxation in Eq. (17) and cost penalization in Eq. (5), however, provide more obvious formulation than those in [41].

9. Charging station' time feasibility and subtour prevention:

$$\tau_{lk} + t_{ljk}x_{ljk} + g_k(\bar{B}_k - r_{lk}) - (l_0 + g_k\bar{B}_k)(1 - x_{ljk}) \leq \tau_{jk}, \forall l \in \mathbb{F}, \\ \forall j \in \mathbb{U}_3, l \neq j, \forall k \in \mathbb{K}. \tag{18}$$

Eq. (18) admits time feasibility for arcs leaving charging station.

10. The relationship between travel time t_{ijk}, distance traveled d_{ij}, and EV speed v_k is given by:

$$t_{ijk} = \frac{d_{ij}}{v_k}, \quad \forall i \in \mathbb{U}_2, \forall j \in \mathbb{U}_3, i \neq j, \forall k \in \mathbb{K}. \tag{19}$$

Eq. (19) is assumed that EV moves at a constant speed.

11. Battery consumption:

$$0 \leq r_{jk} \leq r_{ik} - h_k d_{ij} x_{ijk} + \bar{B}_k(1 - x_{ijk}), \quad \forall i \in \mathbb{V}, \forall j \in \mathbb{U}_3, i \neq j, \forall k \in \mathbb{K}, \tag{20}$$

$$0 \leq r_{jk} \leq \bar{B}_k - h_k d_{lk} x_{ljk}, \quad \forall l \in \mathbb{F}, \forall j \in \mathbb{U}_3, l \neq j, \forall k \in \mathbb{K}. \tag{21}$$

Eq. (20) and Eq. (21) specify the battery level at each customer is determined by the battery consumption and the vehicle's travel.

12. The battery level at the depot is the same as the battery capacity:

$$r_{0k} = \bar{B}_k, \quad \forall k \in \mathbb{K}, \tag{22}$$

$$r_{lk} = \bar{B}_k, \quad \forall l \in \mathbb{F}, \ \forall k \in \mathbb{K}. \tag{23}$$

It is assumed that at the charging station the battery level is at full capacity as stated in Eq. (23).

13. Binary and nonnegativity:

$$x_{ijk} \in \{0,1\}, \quad \forall i \in \mathbb{U}_2, \forall j \in \mathbb{U}_3, \forall k \in \mathbb{K}. \tag{24}$$

In this model, we consider a battery charging station. However, if we allow the operation of a battery swapping station which needs no charging time, then we should set the charging rate $g_k = 0$ and Eq. (18) reduces to

$$\tau_{ik} + t_{ijk}x_{ijk} - b_0(1 - x_{ijk}) \le \tau_{jk}, \quad \forall i \in \mathbb{U}_2, \forall j \in \mathbb{U}_3, i \ne j, \forall k \in \mathbb{K}. \tag{25}$$

The formulation of penalty cost C_p in Eq. (5) obviously makes the optimization problem nonlinear. Thus, to linearize the problem, we reformulate Eq. (5) into the following expression (Eq. (26) – Eq. (29)):

$$C_p = \sum_{i \in \mathbb{N}} \sum_{k \in \mathbb{K}} (c_a v_{ik} + c_b w_{ik}), \tag{26}$$

subject to

$$a_i - \tau_{ik} \le v_{ik}, \quad \forall i \in \mathbb{V}, \ \forall k \in \mathbb{K}, \tag{27}$$

$$\tau_{ik} - b_i \le w_{ik}, \quad \forall i \in \mathbb{V}, \ \forall k \in \mathbb{K}, \tag{28}$$

$$v_{ik}, w_{ik} \ge 0, \quad \forall i \in \mathbb{V}, \ \forall k \in \mathbb{K}. \tag{29}$$

3. Results and Discussion

In this part we demonstrate the applicability of our model by considering simple cases as we solve the model using an exact approach. Two cases, namely homogeneous and heterogenous fleets, are presented. In both cases we consider distribution problems with soft time windows of 1 depot, 5 customers (C1, C2, C3, C4, C5), 2 electric vehicles, and 2 battery charging stations (BCS1, BCS2).

In the first case, we examine a problem with homogeneous fleet, where EVs have the same loading capacity and battery capacity, i.e., $Q_1 = Q_2 = 200$ and $\bar{B}_1 = \bar{B}_2 = 55$. While in the second case, we discuss a heterogeneous fleet distribution problem, where the loading and battery capacities are different: $Q_1 = 200$, $Q_2 = 150$, $\bar{B}_1 = 70$, and $\bar{B}_2 = 55$. Without loss of generality, we assume the EVs have the same velocity $v_1 = v_2 = 1$ km/min and impose the same unit costs (all in rupiahs): fixed cost $c_f = 221{,}448$ per EV, traveling cost $c_t = 2{,}214$ per minute, battery recharging cost $c_r = 66{,}434$ per unit, penalty costs $c_a = c_b = 2{,}214$ per minute. We also set the rates of battery consumption $h_1 = h_2 = 1$ kw/km and the rates of battery recharging $g_1 = g_2 = 1$ kw/min. All distributions should be completed at 08.00 – 13.00, which then converted to 0 – 300 minutes.

The location of customers, their demand q_i, time windows $[a_i, b_i]$, and service time s_i are provided in Table 1. From coordinate of locations, we can calculate the Euclidean distance between two nodes d_{ij}. For soft time windows, we assume that all customers can accept an earliness or tardiness no more than five minutes, i.e., $\rho = 5$.

Table 1. Parameters of model

Index	Nodes	Coordinate	Demand	Time Windows	Service Duration
0	Depot	(5,5)	0	08.00 – 13.00 = [0,300]	0
$1 \in \mathbb{V}$	C1	(35,5)	50	08.10 – 09.50 = [10,110]	34
$2 \in \mathbb{V}$	C2	(17,31)	60	08.15 – 09.44 = [15,164]	47
$3 \in \mathbb{V}$	C3	(37,20)	53	08.28 – 11.50 = [28,230]	54
$4 \in \mathbb{V}$	C4	(35,35)	72	08.34 – 10.34 = [34,154]	28
$5 \in \mathbb{V}$	C5	(4,18)	65	08.18 – 11.00 = [18,180]	30
$1 \in \mathbb{F}$	BCS1	(26,15)	0	08.00 – 13.00 = [0,300]	–
$2 \in \mathbb{F}$	BCS2	(10,34)	0	08.00 – 13.00 = [0,300]	–

The optimal routes obtained from the model using Lingo 17.0 are presented in Fig. 2 and Fig. 3, respectively for the first and second cases. In the figures, a weight assigned to an edge denotes the

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distance between two customers d_{ij} (rounded to nearest integer for simplicity). In the first case where the loading and baterry capacities of EVs are homogeneous, from Fig. 2 we can see that the optimal route for EV1 is Depot–C1–BCS1–C3–C4–BCS2–Depot and that of EV2 is Depot–C5–C2–Depot.

Since we assume that one unit of distance traveled requires one unit of battery, the battery charging status can be verified as follows. EV1 leaves depot for C1 with a full capacity of battery $\bar{B}_1 = 55$. Arriving at C1, which is 30 km away from the depot, the battery power reduces to $55 - 30 = 25$. Arriving at the BCS1 the battery power reduces to $25 - 13 = 12$. After recharging, the battery status returns to 55. Visiting C3 and C4, the battery power consecutively reduces to $55 - 12 = 43$ and $43 - 15 = 28$. The remaining power, i.e., 42, is not sufficient to bring the vehicle back to the depot. Therefore, the vehicle first makes a stop at BCS2 to get a full battery capacity. The journey of EV2 requires no visit to the battery charging station. The total battery consumption, which is equivalent to the total distance traveled, is $13 + 15 + 23 = 51$, lower than the maximum capacity of battery. The total load of EV1 is $q_1 + q_3 + q_4 = 50 + 53 + 72 = 175$, and that of EV2 is $d_5 + d_2 = 65 + 60 = 125$. Both are below the vehicle loading capacities. The left part of Table 2 presents the (hard) time windows and the service times in each visited node. It can be observed that there is a four-minute delivery lateness at C4. The maximum acceptable violation of time windows is five minutes. This tardiness, however, is causing a penalty cost of $4c_b = 4(2{,}214) = 8{,}856$. We also incur fixed costs $2c_f = 2(221{,}448) = 442{,}896$, traveling costs $(124 + 51)c_t = 175(2{,}214) = 387{,}450$, and battery recharging cost $2c_r = 2(66{,}434) = 132{,}868$. All together we obtain the minimum total cost is 972,070. The computation time using standard machine is about 14 seconds.

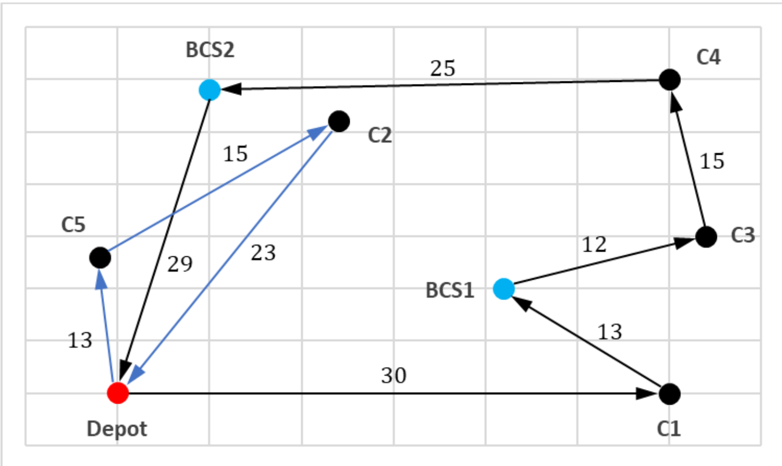

Fig. 2. Optimal routes with homogeneous fleet

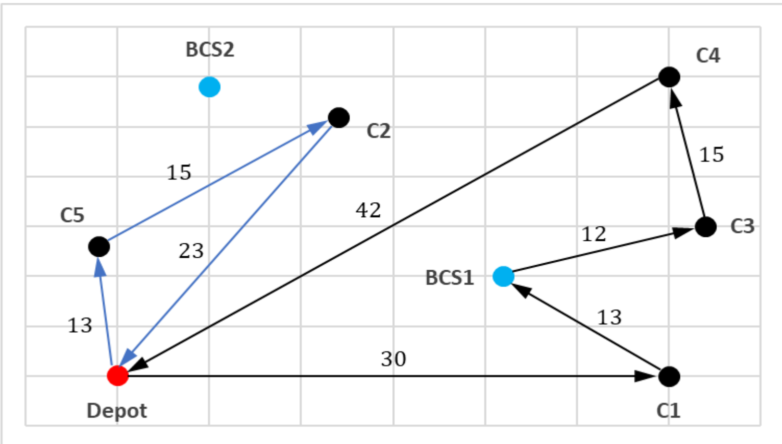

Fig. 3. Optimal routes with heterogeneous fleet

Table 2. Time windows and service time

Homogeneous Fleet				Heterogeneous Fleet			
Vehicle	**Route**	**Time Windows**	**Service Start Time**	**Vehicle**	**Route**	**Time Windows**	**Service Start Time**
EV1	Depot	[0,300]	0	EV1	Depot	[0,300]	0
	C1	[20,110]	30		C1	[20,110]	30
	BCS1	[0,300]	77		BCS1	[0,300]	77
	C3	[28,230]	89		C3	[28,230]	95
	C4	[34,154]	158		C4	[34,154]	154
	BCS2	[0,300]	211		Depot	[0,300]	214
	Depot	[0,300]	242	EV2	Depot	[0,300]	0
EV2	Depot	[0,300]	0		C5	[18,180]	23
	C5	[18,180]	18		C2	[15,164]	85
	C2	[15,164]	63		Depot	[0,300]	128
	Depot	[0,300]	133				

The case of a heterogeneous fleet is a bit challenging as EVs have different loading and battery capacities. The optimal route for EV1 is Depot–C1–BCS1–C3–C4–Depot as illustrated by black arrowed lines in Fig. 3. The battery capacity of EV1 is 70 kw and the battery power consumption is as follows: 70–40–70–29–58–43–1, where the vehicle makes a stop at battery charging station BCS1. The total load of EV1 is $q_1 + q_3 + q_4 = 50 + 53 + 72 = 175$, where the loading capacity of EV1 is $Q_1 = 200$. The optimal route for EV2 is described by blue arrowed lines in Fig. 3, namely Depot–C5–C2–Depot, where the battery power consumption is given by 55–42–27–4 and the total load is $q_5 + q_2 = 65 + 60 = 125$, which is less than the maximum loading capacity of EV2 $Q_2 = 150$. The optimal route of heterogeneous case shares many similarities with that of homogeneous case. The only difference is that EV1 in the heterogeneous case ends the route from C4 directly to the Depot covering 42 km. This can happen because in this case EV1 has now a larger battery capacity, namely 70 kw. The total operational cost is 870,212, cheaper than that om homogeneous case.

Table 2 presents the fulfilment of time windows for homogeneous and heterogeneous cases. For heterogeneous fleet, all deliveries are conducted within the designated (hard) time windows. Thus, no penalty cost is incurred. However, in the homogeneous fleet case, we found a violation of time window of customer C4 by EV1. Even though this violation incurs a penalty cost, soft time windows can be a preferred choice than hard time windows [40, 41], particularly in scenarios where a more practical and robust solution is desired.

4. Conclusion

We have proposed a mixed integer linear programming model for a fleet electric vehicle routing problem with soft time windows. Two simple problems, namely heterogeneous and homogeneous cases, involving one depot, five customers, two electric vehicles, and two battery charging stations are considered and solved using a branch-and-bound method. The mathematical modeling of EVRPs presents several challenging issues. Firstly, EVRPs need to consider not only traditional vehicle routing aspects like minimizing distance or time but also factors specific to EVs, such as battery constraints and charging station availability. Secondly, incorporating realistic and dynamic data on traffic conditions, weather, and energy consumption is complex and can significantly impact route optimization. Thirdly, EVRPs may involve a mixed-integer nonlinear programming (MINLP) formulation, which is computationally intensive and can lead to increased solving times for larger problem instances. Fourthly, the uncertainty surrounding charging station availability and charging times adds an additional layer of complexity and requires robust optimization techniques. Lastly,

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real-world EVRPs must account for practical constraints like pickup-and-delivery and multiple trips, further complicating the modeling process.

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