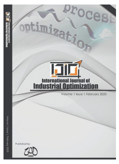
IJIO
(*International Journal of Industrial Optimization*)
ISSN 2723-3022 (online) | 2714-6006 (printed)
Vol. 7, No. 1, 1-14
https://doi.org/10.12928/ijio.v7i1.11340



Effect of return rate on optimal order quantity for single-period products: a modified newsboy problem approach

Wen-Feng Hsiao [a,*], W.C. Tsai [b], Chih-Hsiung Wang [c]

[a] Department of Information Management, National Pingtung University, Taiwan
[b] Department of Industrial Engineering and Management, I-Shou University, Taiwan
[c] Department of Business Administration, National Pingtung University, Taiwan
* Corresponding Author: wfhsiao@mail.nptu.edu.tw

ARTICLE INFO | ABSTRACT

Article history
Received: August 11, 2024
Revised: September 2, 2025
Accepted: October 4, 2025

Keywords
Newsboy model;
Order quantity;
Return rate.

This paper proposes a modified newsboy problem for single-period fashion products, sold by e-stores, for the case where product returns are allowed during the sales season. This is because, even if an e-store sells high-quality products, returns may still occur due to unpredictable consumer behavior in online sales. In the developed model, the assumption from the literature that the number of returns during the sales period is unlimited has been modified to align with real-world conditions. In formulating the problem, the number of returns is assumed to be bounded by a random variable. The corresponding mathematical model is established and the optimal order quantity investigated. The traditional newsboy model is shown to be a special case of the proposed model for which the return factor is ignored. The numerical results show that the optimal order quantity determined when the return rate is ignored is always overestimated and the resulting expected total profit is thus always underestimated. A sensitivity analysis is performed to determine the effects of the model parameters on the optimal solution. The numerical example demonstrates that, in the case of the product return rate, the order quantity predicted by the traditional newsboy model is overestimated by approximately 10%, which results in a profit reduction of about 1%. The numerical results have shown that when the return rate is ignored, the estimated optimal order quantity is higher than the true optimal order quantity and causes the total profit to be slightly underestimated.

1. Introduction

Compared to daily-life products, such as tissue paper, detergent and garbage bags, for example, the ordering (or production) quantity decision-making problem for fashion products, innovative goods, and short life-cycle merchandise is extremely challenging due to their inherently volatile demand [1]. Accordingly, the challenges involved in dealing with such issues as product pricing, order quantity determination, and capacity allocation for such products have attracted significant attention in the literature [2]. Lau and Lau [3] explored the impact of market demand volatility on the pricing policies of the manufacturer (supplier) and retailer (buyer), respectively, and the order quantity of the retailer. Angelus and Porteus [4] constructed a model to facilitate the optimal manufacturing capacity allocation of merchandise with short life cycles. Eppen and Iyer [5] explored the retailer ordering decision-making problem for fashion goods for the case where the supplier agrees to hold a certain fraction of the committed quantity in reserve to mitigate the risk of leftover. Guo and Ma [6] employed the newsboy model to examine the retailer ordering problem for innovative products with an uncertain market demand.

Traditional bricks and mortar stores often operate online stores to facilitate consumer shopping and return transactions [7], [8]. E-stores typically provide merchandise with warranty strategies or loose return policies [9]–[11] enhance the sales of fashion products or similar short life-cycle goods, such as mobile phones and electronic toys [12], [13]. In addition to fashion products, certain products with short life cycles, such as headphones or tablets, may be returned if they fail to meet consumer expectations. As subsequent product style updates are promoted through advertising, these items often need to be disposed of with residual value once the sales season concludes. However, loose return policies not only increase the cost incurred by the seller in processing returns [13], but may also raise the probability of merchandise actually being returned [14], [15]. In fact, a loose return policy may increase the return rate by as much as 35% [16]. Mass returns from online sales present a significant challenge [17]. According to the National Retail Federation, the return rate for online sales in 2023 is projected to be as high as 17.6%. E-commerce companies should implement appropriate measures to mitigate losses caused by returns. To reduce the potentially huge loss stemming from consumer returns, Mukhopadhyay and Setaputra [14] proposed a method for deriving an optimal return policy somewhere between a partial refund and a full refund [18]. Similarly, Hua et al. [19] examined the problem of determining the optimal return fee and shipping fee under an unconditional returns policy.

Li et al. [20] explored the best selling price for advance-selling commodities sold under a full refund or partial refund return policy. Vlachos and Dekker [21], [22] considered various processing strategies for returned goods, including selling to a secondary market, direct re-sales, or repairing to a new state before re-selling to the normal market once again. The authors argued that even when the status of the returned goods is not as good as new, the return rate may still be high if the goods can be re-sold without the need for repair.

In practice, retailers must take into account the return policy provided by the supplier when placing an order on them. For example, Hu et al. [23], [24] showed that consignment contracts between the supplier and the retailer, which allow the retailer to return goods to the manufacturer if left unsold, affects not only the supplier's order decision, but also the manufacturer's production decision. Temur et al. [25] employed a fuzzy expert system to predict the number of returns required to initiate the supplier's reverse logistics activities. Clottey et al. [26] examined various strategies used by manufacturers to incorporate the forecast rate of returns into their production (or reproduction) programs. Chen [27] investigated the effect of the return rate on the manufacturer's pricing decision and retailer's order quantity, respectively, subject to a buyback contract between the two parties [28], [29]. Mostard et al. [30], [31] investigated the optimal order quantity for retailers in the case of fashion goods sold during a single sales season with free returns, where the returns were subsequently repackaged by the supplier and offered for sale once again (refer also to Dey and Chakraborty [32], who described the market demand using fuzzy random variables). Mostard et al. [30] assumed that the goods could be returned without limit during the sales season. By contrast, the present study considers a reformulated newsboy problem in which the number of returns during the sales season is constrained to a certain finite random variable.

Furthermore, while returns may occur primarily due to defects in the product itself [33], [34] or consumer concerns about the received product, such as incorrect size and dissatisfaction with color [35]–[37]. This study focuses on the latter scenario, where the likelihood of a product being returned is treated as probabilistic. The research contributions are as follows: 1. A mathematical model was established, incorporating return rate factors due to non-product defects into the newsboy model to determine the optimal order quantity and maximize corporate profits. 2. It was found that when returns occur, the retailer's order quantity should be smaller than the economic order quantity of the traditional newsboy model. We are also organizing the literature presented in Fig. 1 to analyze how each stage in supply chain management responds to short life cycle products.

 10.12928/ijio.v7i1.11340

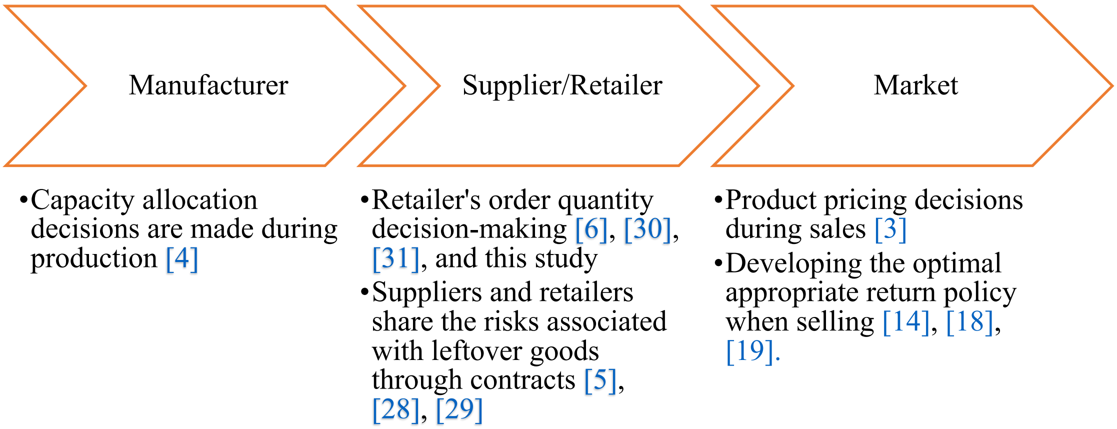


Fig. 1. Solution focus of short life cycle products

The remainder of this paper is organized as follows. Section 2 explains the mathematical model proposed in this study for determining the optimal order quantity of an e-store for fashion products under the revised newsboy problem. Section 3 illustrates the proposed model by means of several numerical examples and investigates the effects of the main model parameters on the optimal order quantity. Finally, Section 4 provides some brief conclusions and indicates the possible direction of future research.

2. Method

The notations used throughout this study in formulating the proposed modified newsboy model are defined as follows:

p_i: probability that the market demand is i, $i = 0,1,2,3,\cdots$;

Q: retailer's order quantity, a decision variable;

c_S: unit cost of stockout for the retailer;

c_P: unit cost of purchase for the retailer;

w: revenue accruing to the retailer from product salvage at the end of the sales season;

R : price per product;

d : cost of processing each returned product, including shipping, repacking, and so on;

k: probability that a returned product is received by the e-store before the end of the sales season;

β: number of product returns during the sales season (equivalent to at most $\beta + 1$ sales opportunities during the sales season), a random finite variable;

$f(\beta)$: probability mass function of β, $\beta = 0,1,2,\cdots$;

v_j: rate of return of the product at its j-th sale (i.e., any physical product may be sold, returned and then re-sold multiple (j) times), $j = 1,2,3,\cdots$, and with $v_0 = 1$;

r_j: expected revenue for the product at its j-th sale, $j = 1,2,\cdots,\beta + 1$, and with $r_{\beta+2} = w$;

When the product is not defective, the behavioral factors driving consumer returns can be complex. For instance, does the practice of including return labels on products encourage more returns, or are consumers engaging in opportunistic returns? (see Table 2) [38]. In response to these considerations, this study explores the sale of products with normal quality, repackaging, and reselling them post-return to maximize corporate profits (see Fig. 2). The mathematical model developed for this study is now introduced as follows.

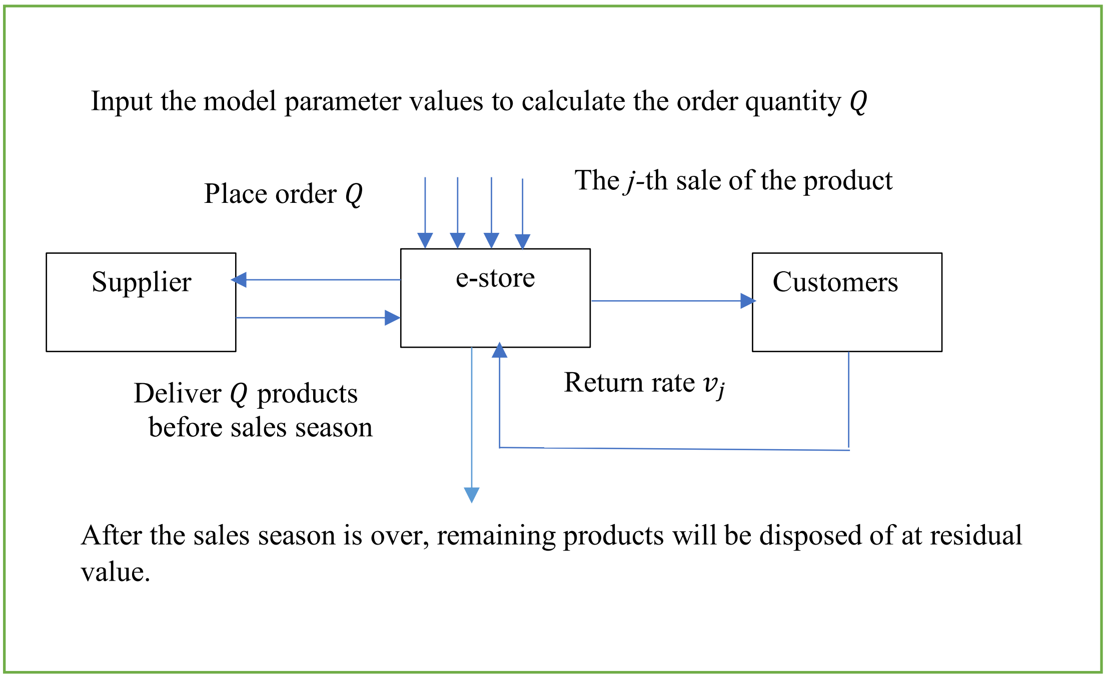


Fig. 2. The revised newsboy model operating diagram

Consider a fashion product with a single sales season, and assume that the e-store retailer wishes to determine the optimal order quantity for this product before the season begins. Ignoring quantity discounts, the purchase cost per product is c_P. For the relationship between purchase cost and purchase quantity, please refer to [39]. Moreover, the unit selling price is R. Assume that products left unsold at the end of the sales season are sold by the retailer at a residual value w, where $0 \leq w \leq c_P < R$. Assume also that when the product is out of stock, a unit opportunity cost of c_S is incurred by the retailer.

Let the gross market demand for the product be i with an associated probability p_i. Products sold by the e-retailer are allowed to be returned for a full refund. If the returned merchandise is received by the e-store before the end of the sales season (with probability k), it can be repackaged and sold once again. Conversely, if the merchandise is received after the end of the sales season (with probability $1-k$), it can be salvaged at a value of w. The cost of processing each returned product is assumed to be d. The remaining assumptions in the proposed ordering model are as follows.

I. The gross supply includes returned products that can be sold once again. In other words, the total number of supplies can exceed the original order amount of the retailer. However, products with fewer returns are sold preferentially to the customers.

II. The probability of return for the product at its j-th sale is denoted by v_j (i.e., the probability of non-return is equal to $1-v_j$), where the three possible cases for v_j are described as follows:

III-i. v_j is non-increasing with j. Such a phenomenon may occur when the retailer continuously upgrades the offered sales service (e.g., by improving the packaging sophistication, shortening the delivery time, or enhancing the on-time delivery rate) such that the customer satisfaction is improved and the return rate correspondingly reduced.

III-ii. v_j is independent of j. In other words, v_j is effectively a constant.

III-iii. v_j is non-decreasing with j. Such a situation may occur if the sold products consistently contain quality, delivery and / or packaging flaws, which degrade customer satisfaction and increase the possibility of returns.

To obtain the expected total profit for a given order quantity, it is necessary to consider the effects of the product return rate on both the gross supply and the revenue accruing to the e-retailer in satisfying the demand. Given an upper bound β on the number of products returned during the sales season, and an original order quantity Q, the maximum amount of gross supply during the sales season is given by

$$Q\left(1+v_1k+v_1v_2k^2+\cdots+\prod_{m=0}^{\beta} v_m\, k^{\beta}\right)=Q\mu_\beta, \tag{1}$$

where $\mu_\beta=\sum_{j=0}^{\beta}\left(\prod_{m=0}^{j} v_m\right)k^j>1$.

As seen in Equation (1), in other words, the supply can be inflated up to $\lfloor Q\mu_\beta\rfloor$. For example, suppose that a product can be returned at most five times during the sales season, that is, $\beta=5$. Under this situation, the product has up to six sales opportunities (i.e., the original sale plus five re-sales). Assume that the return rates for these six sales are as follows: $v_1=0.2$, $v_2=0.36$, $v_3=0.488$, $v_4=0.5904$, $v_5=0.6723$ and $v_6=0.7379$.

The increased percentage in supply is equal to 19.5% when $k=0.7$. In other words, managers must include the impact of the return rate on their ordering decisions if they are to avoid placing excessive orders.

As seen in Equation (2), the average revenue for a product at its j-th sale (denoted by r_j), which includes $j-1$ returns with a total processing cost $(j-1)d$ and the revenue arising from the current sale, can be expressed by the following recursive Equation:

$$r_j=-(j-1)d+R(1-v_j)+v_j\left((1-k)w+kr_{j+1}\right) \text{ for } j=1,2,\cdots,\beta+1,$$

or alternatively as

$$r_j=R-(j-1)d-v_j(R-(1-k)w)+v_jkr_{j+1}. \tag{2}$$

If the $\beta+1$-th sale of a product is still returned (despite the assumed finite maximum number of returns, β), then the $\beta+2$-th sale of the product occurs at the end of the sales season and generates a salvage value of w. In other words, $r_{\beta+2}=w$, and can be regarded as the lower boundary value in obtaining r_j in Equation (2). The following lemma is provided for r_j for $=1,2,\cdots,\beta+1$.

Lemma 1. The expected revenue generated by a product at its j-th sale (i.e., the product has $j-1$ previous returns) is given by

$$r_j=\frac{1}{\prod_{i=1}^{j-1} v_i\, k^{j-1}}\left\{\sum_{i=j-1}^{\beta}\left\{\prod_{i=1}^{j-1} v_i\left\{-(j-1)d+R-v_j(R-(1-k)w)\right\}\right\}+\prod_{i=1}^{\beta+1} v_i\, k^{\beta+1}w\right\} \tag{3}$$

for $j=1,2,\cdots,\beta+1$ and $r_{\beta+2}=w$.

Note that the proof of this lemma is provided in the Appendix A. Given a gross market demand i with corresponding probability p_i for $i=1,2,3,\cdots$, and a maximum number of product returns β, the following discussions determine the optimal order quantity (denoted by Q^*) that maximizes the expected total profit to the retailer. The discussions consider three cases, as outlined in the following:

Case 1. $i\le Q$: When the minimum gross supply (i.e., Q) is greater than the gross market demand i, only i products will be sold and the obtained revenue is thus equal to ir_1. Meanwhile, the number of returned products received by the e-store is equal to iv_1 and, together with the $Q-i$ unsold products, they are salvaged to generate a total revenue of $w(iv_1+Q-i)$.

Case 2. $Q<i\le\lfloor Q\mu_\beta\rfloor$: When the gross market demand i is located between the minimum gross supply amount, Q, and the maximum gross supply amount, $\lfloor Q\mu_\beta\rfloor$, the supply amount will be adjusted to i (or possibly slightly higher. In particular, the retailer searches for the smallest positive integer (denoted by α_i), where $Q\sum_{j=0}^{\alpha_i}\prod_{m=0}^{j} v_m\, k^j\ge i$, which minimally suffices the market

demand i. Note that $\alpha_i \le \beta$ (please refer to Equation (1). In addition, the small surplus amount, $Q\sum_{j=0}^{\alpha_i}\prod_{m=0}^{j} v_m\, k^j - i$, is salvaged to obtain the revenue $w\left(Q\sum_{j=0}^{\alpha_i}(\prod_{m=0}^{j} v_m)k^j - i\right)$. In this case, each product has $\alpha_i + 1$ sales on average given a demand of i, and the resulting expected total revenue is given by $Q(r_1 + v_1 k r_2 + v_1 v_2 k^2 r_3 + \cdots \prod_{m=0}^{\alpha_i} v_m\, k^{\alpha_i} r_{\alpha_i+1}) = Q\sum_{j=0}^{\alpha_i}(\prod_{m=0}^{j} v_m r_{m+1})k^j$.The expected number of returned products that cannot be resold is given by

$$Q\left(v_1(1-k) + v_1 v_2 k(1-k) + v_1 v_2 v_3 k^2(1-k) \ldots + \prod_{m=1}^{\alpha_i} v_m\, k^{\alpha_i - 1}(1-k)\right) = Q\sum_{j=1}^{\alpha_i}(\prod_{m=1}^{j} v_m)k^{j-1}\,(1-k), \tag{4}$$

where these products are salvaged to yield a revenue of $wQ\sum_{j=1}^{\alpha_i}(\prod_{m=1}^{j} v_m)k^{j-1}\,(1-k)$.

Case 3. $\lfloor Q\mu_\beta \rfloor + 1 < i$: When the maximum gross supply is less than the gross market demand i, the expected revenue is given by $Q\left(\sum_{j=0}^{\beta}(\prod_{m=0}^{j} v_m\, r_{m+1})k^j\right)$. Meanwhile, the shortage cost resulting from failing to satisfy the demand is equal to$(i - Q\mu_\beta)c_S$. Finally, the average number of returned products that can be salvaged is given by (refer to Equation (4) and Equation (1)) $Q\sum_{j=1}^{\beta}(\prod_{m=1}^{j} v_m)k^{j-1}\,(1-k) = Q(\mu_\beta - 1)(1/k - 1)$.

Analyzing Cases 1-3 above, the expected total profit (denoted by $g(Q|\beta)$)) for an order quantity Q with a given β is obtained as

$$\begin{aligned} g(Q|\beta) = {} & \sum_{i=1}^{Q} i r_1 p_i + w\sum_{i=1}^{Q}(i v_1 + Q - i)p_i + Q\sum_{i=Q+1}^{\lfloor Q\mu_\beta \rfloor}\left[\sum_{j=0}^{\alpha_i}\left(\prod_{m=0}^{j} v_m r_{m+1}\right)k^j\right]p_i \\ & + w\sum_{i=Q+1}^{\lfloor Q\mu_\beta \rfloor}\left\{Q\sum_{j=1}^{\alpha_i}\left(\prod_{m=1}^{j} v_m\right)k^{j-1}\,(1-k)\right. \\ & \left. + \left[Q\sum_{j=0}^{\alpha_i}\left(\prod_{m=0}^{j} v_m\right)k^j - i\right]\right\}p_i \\ & + Q\sum_{i=\lfloor Q\mu_\beta \rfloor+1}^{\infty}\left[\sum_{j=0}^{\beta}\left(\prod_{m=0}^{j} v_m\, r_{m+1}\right)k^j\right]p_i \\ & + wQ(\mu_\beta - 1)(1/k - 1)\sum_{i=\lfloor Q\mu_\beta \rfloor+1}^{\infty} p_i - c_S\sum_{i=\lfloor Q\mu_\beta \rfloor+1}^{\infty}(i - \lfloor Q\mu_\beta \rfloor)p_i \\ & - c_P Q, \end{aligned} \tag{5}$$

where α_i is the smallest integer that satisfies $Q\sum_{j=0}^{\alpha_i}\prod_{i=0}^{j} v_i\, k^j \ge i$ and $\mu_\beta = \sum_{j=0}^{\beta}(\prod_{m=0}^{j} v_m)k^j$. When the probability distribution of the number of returns for a product is considered, the expected total profit (denoted by $G(Q)$) for an order quantity Q is obtained as $G(Q) = \sum_{\beta=0}^{\infty} g(Q|\beta)f(\beta)$.

Notably, by setting proper parameter values in Equation (3) and Equation (5), namely $f(\beta = 0) = 1$, $v_j = 0$ for $j = 1,2,3,\cdots$, and $v_0 = 1$, the modified newsboy model proposed in this study reduces to the traditional model. One then has $r_1 = R$, $r_2 = w$ and $\alpha_i = 0$. Under this condition, Equation (5) can be rewritten as

$$G(Q) = R\sum_{i=1}^{Q} i p_i + w\sum_{i=1}^{Q}(Q - i)p_i - c_S\sum_{i=Q+1}^{\infty}(i - Q)p_i - c_P Q., \tag{6}$$

Let Q' denote the order quantity that maximizes Equation 6. In other words, Q' can be obtained by satisfying $\sum_{i=1}^{Q'} p_i = \frac{R + c_S - c_P}{R + c_S - w}$ [4].

 10.12928/ijio.v7i1.11340

3. Results and Discussion

A numerical example is first presented to illustrate the proposed modified paperboy model. In formulating the example, the gross market demand is assumed to obey a discrete Weibull distribution [40], that is, $p_i = q^{(i-1)^\gamma} - q^{i^\gamma}$ for $i = 1,2,3,\cdots$, with $q = 0.995$ and $\gamma = 1.6$. The other model parameters are given as follows: $c_S = 1.2$, $c_P = 1$, $w = 0.3$, $R = 3.5$, $d = 0.1$, $k = 0.7$, $\beta = 5$, $f(0) = 0.1$, $f(1) = 0.3$, $f(2) = 0.25$, $f(3) = 0.2$, $f(4) = 0.1$, $f(5) = 0.05$, $v_1 = 0.2$, $v_2 = 0.36$, $v_3 = 0.488$, $v_4 = 0.5904$, $v_5 = 0.6723$ and $v_6 = 0.7379$. From Equation (3), one obtains $r_1 = 3.1997$, $r_2 = 2.7262$, $r_3 = 2.1977$, $r_4 = 1.6447$, $r_5 = 1.1081$, $r_6 = 0.6389$ and $r_7 = w = 0.3$. The average revenue r_j when the product is sold for the j-th time will fall between the selling price and the product's residual value due to the risk of returns with each sale. Based on these r_j, a product liquidation plan can be developed. When returns are not allowed, the remaining products should be liquidated using the discounted selling price. For instance, if a product has been returned $j-1$ times, the sales price should be slightly higher than or equal to r_j.

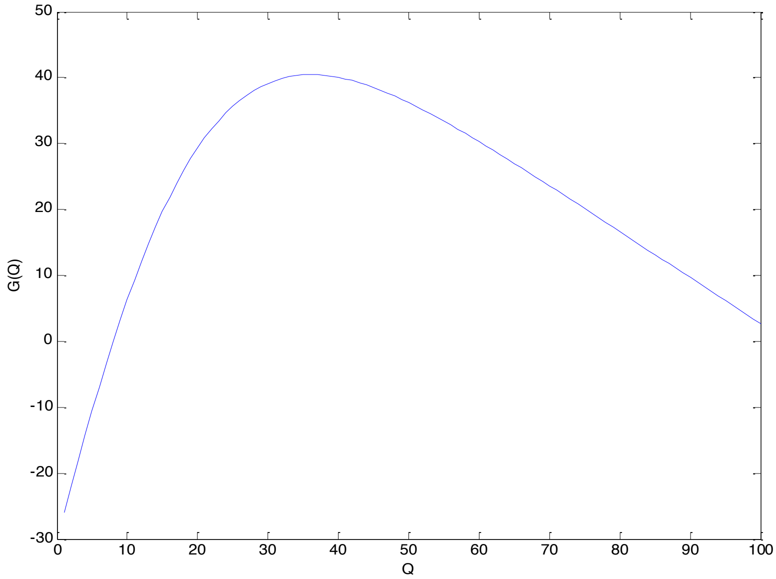


Fig. 3. Change of $G(Q)$ with Q ($q = 0.995$ and $\gamma = 1.6$)

Fig. 3 shows that the optimal order quantity is equal to $Q^* = 35$, for which the expected total revenue has a value of $G(Q^*) = 40.4346$.

Intuitively, when the return rate reduces and the other parameters remain unchanged, both Q^* and the resulting expected total revenue $G(Q^*)$ increase. Conversely, as the return rate increases, Q^* and $G(Q^*)$ reduce. This assertion can be corroborated by the following two examples: when the return rate is larger, that is, $v_1 = 0.35$, $v_2 = 0.5775$, $v_3 = 0.7254$, $v_4 = 0.8215$, $v_5 = 0.884$ and $v_6 = 0.9246$. Fig. 3 shows that $Q^* = 34$ and $G(Q^*) = 32.8261$. Conversely, when the return rate is smaller, i.e., $v_1 = 0.1$, $v_2 = 0.19$, $v_3 = 0.271$, $v_4 = 0.3439$, $v_5 = 0.4095$ and $v_6 = 0.4686$, then $Q^* = 38$ and $G(Q^*) = 42.6824$.

In the following, the proposed model is further employed to evaluate the effects of the Weibull distribution parameters, q and γ, on Q^* given constant settings of all the other model parameters. The results are summarized in Table 1. It is seen that when q increases for constant $\gamma = 1.6$, or γ decreases with constant $q = 0.995$, the demand increases (see also Fig. 4a-4c), which leads in turn to a larger Q^* and $G(Q^*)$. Fig. 4a-4c illustrate the probability distribution of market sales, simulated by adjusting the Weibull function parameter values. The resulting distribution is roughly bell-shaped and right-skewed, indicating that the likelihood of large sales is low in this numerical example. However, in practice, merchants can use past sales data to perform big data analysis to predict the sales probability distribution of products. When applying our proposed model, special attention should be given to estimating return rates, particularly for fashion products that experience rapid changes.

Notably, the optimal order quantity for the traditional newsboy problem can be obtained by setting $Q' = \left\{\ln\left(1 - \frac{R+c_S-c_P}{R+c_S-w}\right)/\ln(q)\right\}^{1/\gamma}$. For example, one has $Q' = 40.06$ and $G(Q') = 40.0439$ when $q = 0.995$ and $\gamma = 1.6$. The difference between the results obtained from the two models for the order quantity and expected total profit can be evaluated as $\Delta_Q\% = 100\% \times \frac{Q'-Q^*}{Q^*}$ and $\Delta_{G(Q)}\% = 100\% \times \frac{G(Q')-G(Q^*)}{G(Q^*)}$, respectively (see Table 1). The corresponding values are found to be $\Delta_Q\% = 10\%$ and $\Delta_{G(Q)}\% = -1\%$, respectively. The difference between the two models can be attributed to the fact that the expected total profit function is insensitive to change around the optimal order quantity (refer to Fig. 3). For the traditional newsboy problem, the optimal order quantity is always overestimated (i.e., $Q' > Q^*$), and hence the total revenue is always underestimated (i.e., $G(Q') < G(Q^*)$), as a result of ignoring the product return rate.

Table 1. Effect of q and γ on optimal solution

$q = 0.995$					$\gamma = 1.6$				
γ	Q^*	$G(Q^*)$	$\Delta_Q\%$	$\Delta_{G(Q)}\%$	q	Q^*	$G(Q^*)$	$\Delta_Q\%$	$\Delta_{G(Q)}\%$
1.3	85	78.51	10.40%	-1.10%	0.99	24	26.54	8.10%	-0.80%
1.4	61	60.92	11.30%	-1.10%	0.991	25	28.31	10.80%	-1.10%
1.5	46	48.93	11.40%	-1.00%	0.992	27	30.42	10.50%	-1.20%
1.6	35	40.43	14.50%	-1.00%	0.993	30	32.93	8.10%	-0.60%
1.7	28	34.21	15.20%	-0.80%	0.994	32	36.2	11.70%	-1.20%
1.8	24	29.52	10.80%	-1.20%	0.995	35	40.43	14.50%	-1.00%
1.9	20	25.92	11.80%	-0.80%	0.996	42	46.38	9.70%	-1.10%
2	17	23.02	12.60%	-0.70%	0.997	50	55.34	10.30%	-1.10%
2.1	15	20.7	10.90%	-1.20%	0.998	64	71.01	11.10%	-1.10%
2.2	13	18.85	12.60%	-1.40%	0.999	99	108.95	10.80%	-1.30%

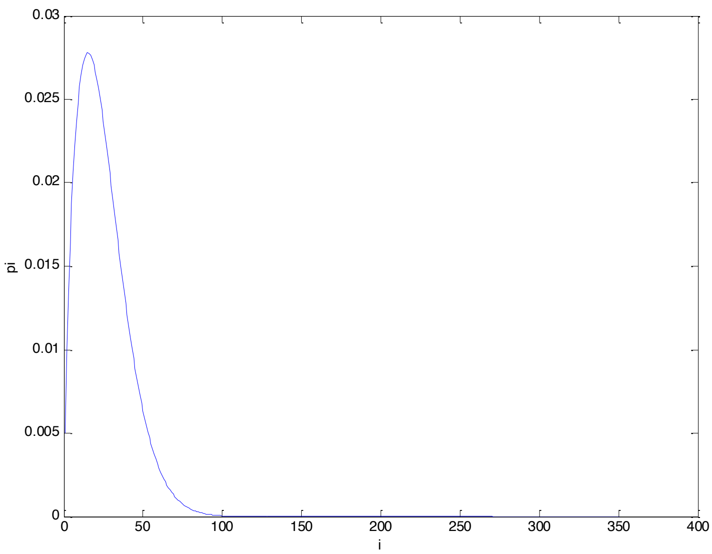


Fig. 4a. Probability distribution of demand with $q = 0.995$ and $\gamma = 1.6$ ($Q^* = 35$ and $G(Q^*) = 40.4369$)

10.12928/ijio.v7i1.11340

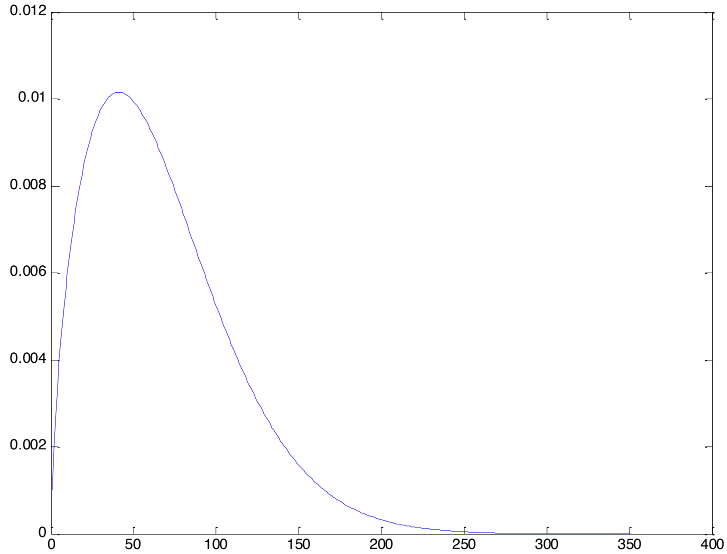


Fig. 4b. Probability distribution of demand with $q = 0.999$ and $\gamma = 1.6$ ($Q^* = 99$ and $G(Q^*) = 108.9454$)

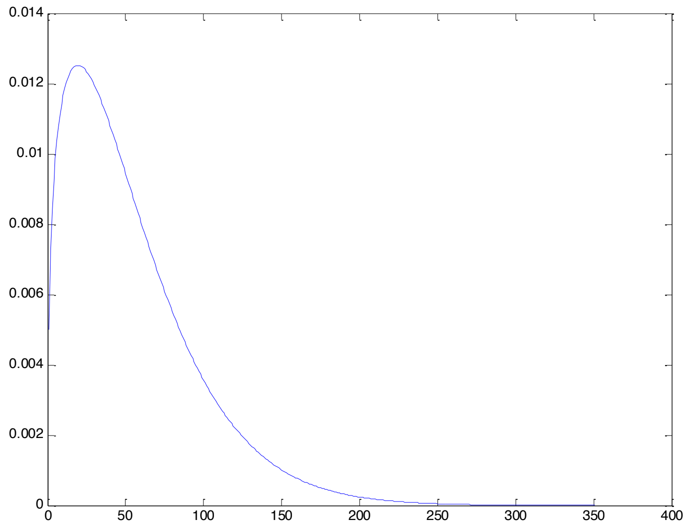


Fig. 4c. Probability distribution of demand with $q = 0.995$ and $\gamma = 1.3$ ($Q^* = 85$, $G(Q^*) = 78.5126$)

Table 2 shows the impact of the cost parameters, c_S, c_P and d, on the optimal order quantity and total profit. When the unit shortage cost c_S increases, the order quantity also increases in order to minimize the stockout cost. The total profit therefore decreases with increasing c_S. When the unit purchase price c_P increases, both the optimal order quantity and the total profit decrease. Finally, as expected, when the unit return processing cost d increases, the total revenue decreases. Furthermore, the optimal order quantity is non-decreasing, albeit the increase is only very slight. This result arises since a larger order quantity avoids the re-sale of products that have already been returned many times, which in turn reduces the chance of return and subsequently lowers the processing cost.

Table 2. Effects of c_S, c_P and d on optimal order quantity and total profit

c_S	Q^*	$G(Q^*)$		c_P	Q^*	$G(Q^*)$		d	Q^*	$G(Q^*)$
0.8	35	41.21		0.6	45	56.51		0	35	41.31
0.9	35	41.02		0.7	42	52.16		0.1	35	40.43
1	35	40.82		0.8	40	48.04		0.2	37	39.66
1.1	35	40.63		0.9	38	44.15		0.3	37	38.95
1.2	35	40.43		1	35	40.43		0.4	37	38.28
1.3	37	40.26		1.1	35	36.93		0.5	37	37.63
1.4	37	40.1		1.2	33	33.55		0.6	37	37.01
1.5	37	39.94		1.3	32	30.29		0.7	37	36.4
1.6	37	39.78		1.4	31	27.15		0.8	37	35.81
1.7	38	39.62		1.5	30	24.12		0.9	37	35.23

Table 3. Effects of R and w on optimal order quantity and total profit

R	Q^*	$G(Q^*)$		w	Q^*	$G(Q^*)$
3	35	29.92		0	33	34.9
3.1	35	32.02		0.1	34	36.67
3.2	35	34.12		0.2	35	38.52
3.3	35	36.22		0.3	35	40.43
3.4	35	38.33		0.4	38	42.53
3.5	35	40.43		0.5	39	44.76
3.6	37	42.56		0.6	42	47.18
3.7	37	44.7		0.7	45	49.79
3.8	37	46.84		0.8	48	52.72
3.9	37	48.98		0.9	54	56.08
4	37	51.13		1	90	60.51

Table 3 shows the effects of R and w on the optimal order quantity and total revenue. As the unit price R increases, the expected total profit also increases. The optimal order quantity also increases slightly. In addition, a larger unit salvage value w increases both the optimal order quantity and the total profit as the result of a lower leftover cost. It is noted that the unit salvage value should be lower-bounded by c_P. For the case of $w = c_P$, when the order quantity exceeds 90, the total profit is almost saturated (see Fig. 5) due to the finite market demand. In comparison with previous literature, Mostard et al. [30] assumed that the number of returns during the sales season is infinite, which contradicts common sense. Therefore, this study modified the assumption to a finite number of returns. Cheng et al. [34] posited that returns only occur when goods are defective, equating the return rate with the defective rate. However, this study considers that returns occur due to consumer dissatisfaction, regardless of whether the goods are defective or not. Cheng et al. [34] also found that an increase in the return rate reduces the retailer's order volume and profits, a finding consistent with the study's results. However, this study further examines the impact of stockout costs on order quantities, an aspect not addressed in Cheng et al.'s [34] model. According to the proposed model, underestimating or ignoring stockout costs will lead to ordering quantities below the optimal level.

 10.12928/ijio.v7i1.11340

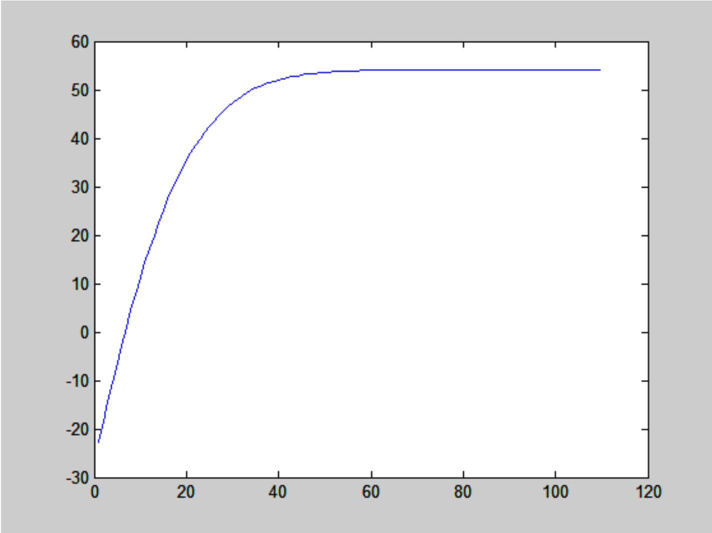


Fig. 5. Change of $G(Q)$ with Q when $w = c_P = 1$

4. Conclusion

This study has proposed a modified newsboy model for determining the optimal order quantity of an e-retailer for single-period products with a non-negligible return rate and a finite random number of returns during the sales season. The numerical results have shown that when the return rate is ignored (i.e., as in the traditional newsboy model), the estimated optimal order quantity is higher than the true optimal order quantity and causes the total profit to be slightly underestimated. In the modified model proposed in this study, the probability that a returned product is received by the e-store before the end of the sales season can be easily extended to be dependent on the number of product returns (see Appendix B). Future research should focus on expanding this model to address the returns of defective products (such as product damage during production or distribution) and the returns of good products due to consumer-related factors.

Author Contribution: All authors contributed equally to the main contributor to this paper. All authors read and approved the final paper.

Funding: This research received no external funding.

Acknowledgment: We thank the Directorate of Research and Community Service and the Department of Industrial Engineering, Department of Information Management and Department of Business Administration.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Proof of Lemma

From Equation (2), for $j = 1,2,\cdots,\beta+1$, the formulae for r_j can be given explicitly, as follows:

$$r_1 = R - v_1(R - (1-k)w) + v_1 k r_2,$$

$$v_1 k r_2 = v_1 k\{-d + R - v_2(R + 2d - (1-k)w)\} + v_1 v_2 k^2 r_3,$$

$$v_1 v_2 k^2 r_3 = v_1 v_2 k^2\{-2d + R - v_3(R + d - (1-k)w)\} + v_1 v_2 v_3 k^3 r_4,$$

$$\dots$$

$$\prod_{i=1}^{j-1} v_i\, k^{j-1} r_j = \prod_{i=1}^{j-1} v_i \{-(j-1)d + R - v_j(R - (1-k)w)\} + \prod_{i=1}^{j} v_i\, k^j r_{j+1},$$

…,

$\prod_{i=1}^{\beta} v_i\, k^{\beta} r_{\beta+1} = \prod_{i=1}^{\beta} v_i\, k^{\beta}\{-\beta d + R - v_{\beta+1}(R + (\beta+1)d - (1-k)w)\} + \prod_{i=1}^{\beta+1} v_i\, k^{\beta+1} r_{\beta+2}$ and

$\prod_{i=1}^{\beta+1} v_i\, k^{\beta+1} r_{\beta+2} = \prod_{i=1}^{\beta+1} v_i\, k^{\beta+1} w$, respectively.

Aggregating the terms from the j-th equation to the final equation gives $\prod_{i=1}^{j-1} v_i\, k^{j-1} r_j = \sum_{i=j-1}^{\beta}\left\{\prod_{i=1}^{j-1} v_i\{-(j-1)d + R - v_j(R-(1-k)w)\}\right\} + \prod_{i=1}^{\beta+1} v_i\, k^{\beta+1} w$, for $j = 1,2,\cdots,\beta+1$.

The last equation can be easily rearranged as Equation (3).

Appendix B

That is, term $v_j k$ in Equation (5) can be replaced with $v_j k_j$, where k_j denotes the probability that the product is received by the e-store before the end of the sales season given that the product is returned after its ith sale. For example, in Equation (1), given β and an order quantity Q, the maximum gross supply during the sales season can be rewritten as

$$Q\left(1 + v_1 k_1 + v_1 v_2 k_1 k_2 + \cdots + \prod_{m=0}^{\beta} v_m\, k_m\right) = Q \sum_{j=0}^{\beta} \prod_{m=0}^{j} v_m\, k_m,$$

with Equation (3) then rewritten as

$$r_j = \frac{1}{\prod_{i=1}^{j-1} v_i\, k_i}\left\{\sum_{i=j-1}^{\beta}\left\{\prod_{i=1}^{j-1} v_i\{-(j-1)d + R - v_j(R-(1-k_j)w)\}\right\} + \prod_{i=1}^{\beta+1} v_i\, k_i w\right\}.$$

References

[1] C. Canyakmaz, S. Özekici, and F. Karaesmen, "A newsvendor problem with markup pricing in the presence of within-period price fluctuations," *Eur. J. Oper. Res.*, vol. 301, no. 1, pp. 153–162, Aug. 2022, doi: 10.1016/j.ejor.2021.09.042.

[2] A. Khazaei and P. Samouei, "Newsboy Problem with Outsourcing, Limited Capacity and Resalable Return.," *J. Decis. &Operations Research*, 2023, [Online]. Available: https://doi.org/10.22105/dmor.2021.266283.1298

[3] A. Hing Ling Lau and H.-S. Lau, "The effects of reducing demand uncertainty in a manufacturer–retailer channel for single-period products," *Comput. Oper. Res.*, vol. 29, no. 11, pp. 1583–1602, Sep. 2002, doi: 10.1016/S0305-0548(01)00047-8.

[4] A. Angelus and E. L. Porteus, "Simultaneous Capacity and Production Management of Short-Life-Cycle, Produce-to-Stock Goods Under Stochastic Demand," *Manage. Sci.*, vol. 48, no. 3, pp. 399–413, Mar. 2002, doi: 10.1287/mnsc.48.3.399.7726.

[5] G. D. Eppen and A. V. Iyer, "Backup Agreements in Fashion Buying—The Value of Upstream Flexibility," *Manage. Sci.*, vol. 43, no. 11, pp. 1469–1484, Nov. 1997, doi: 10.1287/mnsc.43.11.1469.

[6] P. Guo and X. Ma, "Newsvendor models for innovative products with one-shot decision theory," *Eur. J. Oper. Res.*, vol. 239, no. 2, pp. 523–536, Dec. 2014, doi: 10.1016/j.ejor.2014.05.028.

[7] E. Widodo, K. Takahashi, K. Morikawa, I. N. Pujawan, and B. Santosa, "Managing sales return in dual sales channel: its product substitution and return channel analysis," *Int. J. Ind. Syst. Eng.*, vol. 9, no. 1, p. 67, 2011, doi: 10.1504/IJISE.2011.042539.

[8] S. Yan, X. Xu, and Y. Bian, "Pricing and Return Strategy: Whether to Adopt a Cross-Channel Return Option?," *IEEE Trans. Syst. Man, Cybern. Syst.*, vol. 50, no. 12, pp. 5058–5073, Dec. 2020, doi: 10.1109/TSMC.2020.2964560.

[9] S. Wu, "Warranty return policies for products with unknown claim causes and their optimisation," *Int. J. Prod. Econ.*, vol. 156, pp. 52–61, Oct. 2014, doi: 10.1016/j.ijpe.2014.05.016.

[10] W. Pan and C. H. Huynh, "Optimal operational strategies for online retailers with demand and return uncertainty," *Oper. Manag. Res.*, vol. 16, no. 2, pp. 755–767, Jun. 2023, doi: 10.1007/s12063-022-00321-4.

[11] A. H. M. Mashud, H.-M. Wee, C.-V. Huang, and J.-Z. Wu, "Optimal Replenishment Policy for Deteriorating Products in a Newsboy Problem with Multiple Just-in-Time Deliveries," *Mathematics*, vol. 8, no. 11, p. 1981, Nov. 2020, doi: 10.3390/math8111981.

[12] T. Higuchi and M. D. Troutt, "Dynamic simulation of the supply chain for a short life cycle product—Lessons from the Tamagotchi case," *Comput. Oper. Res.*, vol. 31, no. 7, pp. 1097–1114, Jun. 2004, doi: 10.1016/S0305-0548(03)00067-4.

[13] B. Lantz and K. Hjort, "Real e-customer behavioural responses to free delivery and free returns," *Electron. Commer. Res.*, vol. 13, no. 2, pp. 183–198, May 2013, doi: 10.1007/s10660-013-9125-0.

[14] S. K. Mukhopadhyay and R. Setoputro, "Reverse logistics in e-business," *Int. J. Phys. Distrib. Logist. Manag.*, vol. 34, no. 1, pp. 70–89, Jan. 2004, doi: 10.1108/09600030410515691.

[15] S. K. Mukhopadhyay and R. Setaputra, "A dynamic model for optimal design quality and return policies," *Eur. J. Oper. Res.*, vol. 180, no. 3, pp. 1144–1154, Aug. 2007, doi: 10.1016/j.ejor.2006.05.016.

[16] H. Yang, J. Chen, X. Chen, and B. Chen, "The impact of customer returns in a supply chain with a common retailer," *Eur. J. Oper. Res.*, vol. 256, no. 1, pp. 139–150, Jan. 2017, doi: 10.1016/j.ejor.2016.06.011.

[17] L. Shen, F. Lin, Y. Wang, L. Ding, T. C. E. Cheng, and D. Wang, "Decision and Coordination of an E-commerce Supply Chain Considering Returns and Network Externalities," *J. Syst. Sci. Syst. Eng.*, vol. 33, no. 5, pp. 552–575, Oct. 2024, doi: 10.1007/s11518-024-5609-9.

[18] J. Chen and P. C. Bell, "Implementing market segmentation using full-refund and no-refund customer returns policies in a dual-channel supply chain structure," *Int. J. Prod. Econ.*, vol. 136, no. 1, pp. 56–66, Mar. 2012, doi: 10.1016/j.ijpe.2011.09.009.

[19] Z. Hua, H. Hou, and Y. Bian, "Optimal Shipping Strategy and Return Service Charge Under No-Reason Return Policy in Online Retailing," *IEEE Trans. Syst. Man, Cybern. Syst.*, vol. 47, no. 12, pp. 3189–3206, Dec. 2017, doi: 10.1109/TSMC.2016.2564920.

[20] Y. Li, L. Xu, T.-M. Choi, and K. Govindan, "Optimal Advance-Selling Strategy for Fashionable Products With Opportunistic Consumers Returns," *IEEE Trans. Syst. Man, Cybern. Syst.*, vol. 44, no. 7, pp. 938–952, Jul. 2014, doi: 10.1109/TSMC.2013.2280558.

[21] D. Vlachos and R. Dekker, "Return handling options and order quantities for single period products," *Eur. J. Oper. Res.*, vol. 151, no. 1, pp. 38–52, Nov. 2003, doi: 10.1016/S0377-2217(02)00596-9.

[22] G. Ni, "Optimal decisions on price and inventory for a newsboy-type retailer with identifiable information and discount promotion," *PLoS One*, vol. 18, no. 7, p. e0288874, Jul. 2023, doi: 10.1371/journal.pone.0288874.

[23] W. Hu, Y. Li, and K. Govindan, "The impact of consumer returns policies on consignment contracts with inventory control," *Eur. J. Oper. Res.*, vol. 233, no. 2, pp. 398–407, Mar. 2014, doi: 10.1016/j.ejor.2013.03.015.

[24] M. Bieniek, "Consumer returns in consignment contracts with inventory control and additive uncertainty," *INFOR Inf. Syst. Oper. Res.*, vol. 59, no. 1, pp. 169–189, Jan. 2021, doi: 10.1080/03155986.2020.1796065.

[25] G. Tekin Temur, M. Balcilar, and B. Bolat, “A fuzzy expert system design for forecasting return quantity in reverse logistics network,” *J. Enterp. Inf. Manag.*, vol. 27, no. 3, pp. 316–328, Apr. 2014, doi: 10.1108/JEIM-12-2013-0089.

[26] T. Clottey, W. C. Benton, and R. Srivastava, “Forecasting Product Returns for Remanufacturing Operations,” *Decis. Sci.*, vol. 43, no. 4, pp. 589–614, Aug. 2012, doi: 10.1111/j.1540-5915.2012.00362.x.

[27] J. Chen, “The impact of sharing customer returns information in a supply chain with and without a buyback policy,” *Eur. J. Oper. Res.*, vol. 213, no. 3, pp. 478–488, Sep. 2011, doi: 10.1016/j.ejor.2011.03.027.

[28] J. Chen and P. C. Bell, “The impact of customer returns on pricing and order decisions,” *Eur. J. Oper. Res.*, vol. 195, no. 1, pp. 280–295, May 2009, doi: 10.1016/j.ejor.2008.01.030.

[29] Y. Saito and E. Kusukawa, “Impact of Information Sharing Regarding Customer Returns Ratio on Optimal Sales Strategy under E-commerce,” *Ind. Eng. Manag. Syst.*, vol. 14, no. 2, pp. 111–121, Jun. 2015, doi: 10.7232/iems.2015.14.2.111.

[30] J. Mostard, R. de Koster, and R. Teunter, “The distribution-free newsboy problem with resalable returns,” *Int. J. Prod. Econ.*, vol. 97, no. 3, pp. 329–342, Sep. 2005, doi: 10.1016/j.ijpe.2004.09.003.

[31] J. Mostard and R. Teunter, “The newsboy problem with resalable returns: A single period model and case study,” *Eur. J. Oper. Res.*, vol. 169, no. 1, pp. 81–96, Feb. 2006, doi: 10.1016/j.ejor.2004.04.048.

[32] O. Dey and D. Chakraborty, “A single period inventory problem with resalable returns: A fuzzy stochastic approach,” *International Journal of Mathematical*. 2008, [Online].

[33] H. Ren, R. Chen, and Z. Lin, “A Study of Electronic Product Supply Chain Decisions Considering Quality Control and Cross-Channel Returns,” *Sustainability*, vol. 15, no. 16, p. 12304, Aug. 2023, doi: 10.3390/su151612304.

[34] F. Cheng, J. Zhang, and L. Xue, “Study on The Optimal Order Quantity of single-period products Returned for Continuous Demand,” 2017, doi: 10.2991/iccse-17.2017.41.

[35] B. M. Brand and C. S. Kopplin, “Effective Return Prevention Measures in the Post-purchase Stage: A Best-Worst Scaling Approach,” *Mark. ZFP*, vol. 45, no. 1, pp. 30–47, 2023, doi: 10.15358/0344-1369-2023-1-30.

[36] T. Joshi, A. Mukherjee, and G. Ippadi, “One Size Does Not Fit All: Predicting Product Returns in E-Commerce Platforms,” in *2018 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining (ASONAM)*, Aug. 2018, pp. 926–927, doi: 10.1109/ASONAM.2018.8508486.

[37] A. Nestler, N. Karessli, K. Hajjar, R. Weffer, and R. Shirvany, “SizeFlags: Reducing Size and Fit Related Returns in Fashion E-Commerce,” in *Proceedings of the 27th ACM SIGKDD Conference on Knowledge Discovery & Data Mining*, Aug. 2021, pp. 3432–3440, doi: 10.1145/3447548.3467160.

[38] T. S. Robertson, R. Hamilton, and S. D. Jap, “Many (Un)happy Returns? The Changing Nature of Retail Product Returns and Future Research Directions,” *J. Retail.*, vol. 96, no. 2, pp. 172–177, Jun. 2020, doi: 10.1016/j.jretai.2020.04.001.

[39] H. LIU, “Retailer’s Optimal Ordering Decision Based on Cost Curve,” 2021, doi: 10.2991/assehr.k.210806.022.

[40] T. Nakagawa and S. Osaki, “The Discrete Weibull Distribution,” *IEEE Trans. Reliab.*, vol. R-24, no. 5, pp. 300–301, Dec. 1975, doi: 10.1109/TR.1975.5214915.