Bulletin of Applied Mathematics and Mathematics Education
Volume 2, Number 1, April 2022 pp. 19-32
https://dx.doi.org/10.12928/bamme.v2i1.3953
e-ISSN 2776-1029, p-ISSN 2776-1002

Fuzzy regression model with Bayesian approach and its application to public health data

Wan Muhamad Amir Wan Ahmad[1*], Nurul Asyikin Nizam Akbar[2], Farah Muna Mohamad Ghazali[1], Nor Farid Mohd Noor[1], Nor Azlida Aleng[3]

[1]School of Dental Sciences, Health Campus, Universiti Sains Malaysia, 16150 Kubang Kerian, Kota Bharu, Kelantan, Malaysia
[2]Department of Hematology & Transfusion Medicine Unit, Health Campus, Universiti Sains Malaysia, 16150 Kubang Kerian, Kota Bharu, Kelantan, Malaysia
[3]Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, Kuala Terengganu, Terengganu, Malaysia.

*Corresponding E-mail: wmamir@usm.my

ARTICLE INFO

Article History
Received 11 April 2021
Revised 20 January 2022
Accepted 1 May 2022

Keywords
Bootstrap
Bayesian linear regression
Fuzzy regression

How to cite this article:
Wan Ahmad, W. M., Akbar, N. A. N., Ghazali, F. M. M., Mohd Noor, N. F., & Aleng, N. A. (2022). Fuzzy regression model with Bayesian approach and its application to public health data. *Bulletin of Applied Mathematics and Mathematics Education*, *2*(1), 19-32.

ABSTRACT

The application of the Bayesian Linear Regression (BLR) and Fuzzy Bayesian Linear Regression method through the SAS algorithm is the focus of this paper. As an alternative method of data analysis in biostatistics, this modified method can be used. This modified method includes a bootstrapping technique, residual normality checking and some Bayesian Linear Regression Modeling (BLR) enhancement through Fuzzy Bayesian Linear Regression. We illustrated the application of the algorithm for Bayesian Linear Regression (BLR) and Fuzzy Bayesian Linear Regression in this paper.

Introduction

Bayesian Linear Regression (BLR) analysis is an approach to linear regression in which the statistical analysis is undertaken within the context of Bayesian inference. This technique can be applied to forecast the value of the response variables (dependent) when given any value of the predictor variables (independent variables). A general regression model is given by $y_i = E(y_i \mid x_i) + e_i$, where i =1,2,3,…,n denoting an observation of a subject. y_i is the response variable and x_i is a $k \times 1$ vector of independent variables. $E(y_i \mid x_i)$ is the expectation of y_i conditional on x_i, and e_i is the error term. This paper provides an algorithm for Bayesian Multiple Linear Regressions (BMLR) in SAS (Diem Ngo & La Puente, 2012).

Assume a BLR model where the response vector y has dimension $n \times 1$ and follows a multivariate Gaussian distribution with mean $X\beta$ and covariance matrix $\sigma^2 I$, where X the design matrix has dimension $n \times p$, β contains the p regression coefficients, σ^2 is the common variance of the observational and I is an $n \times n$ identity matrix. That is, $y \sim N(X\beta, \sigma^2)$. In the Bayesian approach, the data are supplemented with additional information in the form of a prior probability distribution. The prior belief about the parameters is combined with the data's likelihood function according to Bayes theorem to yield the posterior belief about the parameters β and σ (Gelman et al., 2013; Gelman & Hill, 2006). Data transformation + tools are commonly used to improve the normality of distribution and equalizing variance to meet assumptions and improve effect sizes, thus constituting important aspects of data cleaning and preparing for statistical analyses. The traditional transformations that are commonly discussed include: adding constants, square root, converting to logarithmic scales, inverting and reflecting, and applying trigonometric transformations such as sine wave transformations (Osborne, 2010). The study uses Box-Cox transformation. The form of Box-Cox transformation is as below.

$$y(\lambda) = \begin{cases} \dfrac{y^{\lambda} - 1}{\lambda}, & if \ \lambda \neq 0 \\ \log y & if \ \lambda = 0 \end{cases}$$

where, y is the observation data and λ is the model parameter. The optimal value of λ was determined and this study used, λ = 2. The example for the application of the method discussed by using SAS language computer software is provided (Osborne, 2010).

The bootstrap methods begin with original data or sample that is taken from the population, then calculated as sample statistics. The next step is to copy the original sample several times to create a pseudo-population with replacement by using the empirical density function (EDF), (Efron, Bradley and Tibshirani, 1993). The benefit of using bootstrap is its capability to develop a sample the same size of the original, which may include an observation several times while omitting other observations. The bootstrap method draws the samples with replacement and calculates statistics for each sample (it stores these statistics and creates a distribution for further analysis). After finalizing the bootstrap, the data is analyzed for mean, standard deviation, confidence intervals, and any other evidence of replication (Cassel, 2010; Jung, Jhun, & Lee, 2005; Higgins, 2005). In applying the bootstrap method, the original findings from the empirical test were replicated several times to meet the research requirement. As an example, for 1000 observations (original data), the analysis is performed by using a statistical linear model. The analysis results of beta coefficients and r-squared are obtained, followed by the application of the bootstrapping method to the selected data. In applying the bootstrap method, a sample of 23 observations was replicated 6 times (this is equal to 115 observations). The analysis from the statistical linear model, the beta coefficients, and r-squared values of the bootstrap method was compared to the original results. The bootstrap method findings depict the average beta coefficients and r-squared values that are similar to the original findings, from where it was replicated. Interestingly, the bootstrap method provides another noble opportunity for a further comprehensive study of science and non-science discipline. A fuzzy regression model can be written as $Y = Z_0 + Z_1x_1 + Z_2x_2 + \ldots + Z_kx_k$, here the explanation variables x_i's are assumed to be precise. However, according to the equation above, response variable Y is not crisp but is fuzzy, the same which also applies to the parameters. We aim to estimate these parameters. In further discussion, Z_i's are assumed as symmetric fuzzy numbers

10.12928/bamme.v2i1.3953

which can be presented by intervals. For example, Z_i can be expressed as a fuzzy set given by $Z_i =< a_{1c}, a_{1w} >$ where a_{ic} is center and a_{iw} is radius or vagueness associated. The fuzzy set above reflects the confidence in the regression coefficients around a_{ic} in terms of symmetric triangular membership functions. The application of this method should be given more attention when the underlying phenomenon is fuzzy, which indicates that the response variable is fuzzy. T, the relationship is also considered to be fuzzy. This $Z_i =< a_{1c}, a_{1w} >$ can be written as $Z_1 = [a_{1L}, a_{1R}]$ with $a_{1L} = a_{1c} - a_{1w}$ and $a_{1R} = a_{1c} - a_{1w}$. In fuzzy regression methodology, parameters are estimated by minimizing total vagueness in the model. $y_j = Z_0 + Z_1 x_{1j} + Z_2 x_{2j} + \ldots + Z_k x_{kj}$ .Using $Z_i =< a_{1c}, a_{1w} >$,it can be written $y_j =< a_{0c}, a_{0w} > + < a_{1c}, a_{1w} > x_{1j} + \ldots + < a_{nc}, a_{nw} > x_{nj}$. $=< a_{jc}, a_{jw} >$. Thus, this can be written as $y_{jc} = a_{0c} + a_{1c} x_{1j} + \cdots + a_{nc} x_{nj}$ then it can be written directly as $y_{jw} = a_{0w} + a_{1w} \left| x_{1j} \right| + \cdots + a_{nw} \left| x_{nj} \right|$. y_{jw} represents radius and cannot be negative, therefore, on the right-hand side of equation $y_{jw} = a_{0w} + a_{1w} \left| x_{1j} \right| + \cdots + a_{nw} \left| x_{nj} \right|$, absolute values of x_{ij} are taken. Suppose, there m data point, each comprising $a(n+1) - row$ vector. Then parameters Z_i are estimated by minimizing the quantity, which is the total vagueness of the model-data set combination, subject to the constraint that each data point must fall within the estimated value of the response variable. This can be visualized as the following linear programming problem, minimized $\sum_{j=1}^{m} \left(a_{0w} + a_{1w} \left| x_{1j} \right| + \cdots + a_{nw} \left| x_{nj} \right| \right)$ and subject to

$$\left\{ \left(a_{0c} + \sum_{i=1}^{n} a_{ic} x_{ij} \right) + \left(a_{0w} + \sum_{i=1}^{n} a_{iw} x_{ij} \right) \right\} \geq Y_j$$

and

$$\left\{ \left(a_{0c} + \sum_{i=1}^{n} a_{ic} x_{ij} \right) - \left(a_{0w} + \sum_{i=1}^{n} a_{iw} x_{ij} \right) \right\} \leq Y_j.$$

and $a_{iw} \geq 0$. The simple procedure is commonly used to solve the linear programming problem. (Kacprzyk & Fedrizzi, 1992). Data for this study is a sample which is composed of four variables.

Method

Sample size determination

The sample size for multiple regression analysis was calculated by using G*power with effect size = 0.15, 0.05, power of the study = 0.80, and the number of predictors were three. The minimum sample size requires is 77 respondents.

Analysis:	A priori: Compute required sample size	
Input:	Effect size f^2	= 0.15
	α err prob	= 0.05
	Power (1-β err prob)	= 0.80
	Number of predictors	= 3
Output:	Noncentrality parameter λ	=11.55000
	Critical F	= 2.730
	Numerator, df.	= 3
	Denominator, df.	= 73
	Total sample size	= 77
	Actual power	= 0.80

We used variables as shown in Table 1.

Table 1. Description of cholesterol data

Num.	Variables	Explanation of user variables
1.	Choltot	Total Cholesterol
2.	Hdl	HDL Cholesterol
3	Trig	Triglycerides
4	Waist	Waist circumferences

Algorithm and flow chart for modified Bayesian linear regression analysis method
The algorithm for modified Bayesian linear regression analysis method is presented in Figure 1.

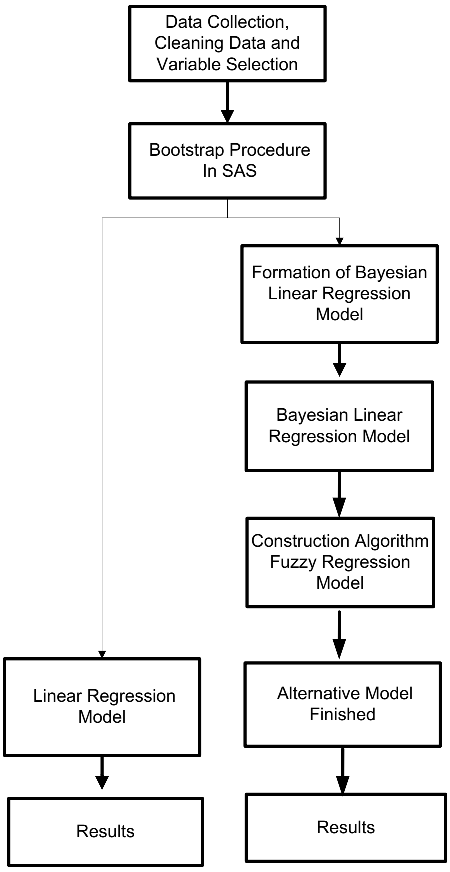

Figure 1. Modified Bayesian linear regression analysis

```
/*ADDING BOOTSTRAPPING ALGORITHM TO THE METHOD */
%MACRO bootstrap(data=_last_, booted=booted, boots=2, seed=1234);
    DATA &booted;
        ** randomly picks an integer from 1 to n;
        pickobs = INT(RANUNI(&seed)*n)+1;
        ** POINT tells SAS to read value pickobs
        ** NOBS sets n to number of obs in &Data;
        ** when the point option is used SAS will loop through the data step
forever;
        SET &data POINT = pickobs NOBS = n;
        ** saves number of current bootstrap;
        REPLICATE=int(i/n)+1;
        i+1;
```


10.12928/bamme.v2i1.3953

```
        ** stop will leave data set when n*&boots obs have been created;
        IF i > n*&boots THEN STOP;
        RUN;

%MEND bootstrap;

Data Cholesterol;
Input choltot Hdl Trig Waist;
Cards;
181   46    76     98.0
220   39    151    94.0
220   39    151    94.0
213   45    123    95.0
179   42    139    81.0
179   42    139    81.0
114   42    62    104.0
114   42    62    104.0
267   71    122    91.5
267   71    122    91.5
235   73    91     96.5
247   55    85     92.0
199   57    126   116.5
199   57    126   116.5
162   45    100    88.0
237   70    222    91.5
207   66    81     85.0
202   49    118   106.5
184   43    98     90.0
299   56    207   113.0
184   47    118    95.0
181   92    71     97.5
220   39    151    94.0
220   39    151    94.0
180   47    58     97.0
179   42    139    81.0
179   42    139    81.0
179   42    139    81.0
179   42    139    81.0
114   42    62    104.0
114   42    62    104.0
114   42    62    104.0
247   55    85     92.0
199   57    126   116.5
199   57    126   116.5
162   45    100    88.0
210   32    193    95.5
237   70    222    91.5
207   66    81     85.0
202   49    118   106.5
299   56    207   113.0
184   47    118    95.0
;
ods rtf file='abc.rtf' style=journal;

**generate bootstrap sample;

%bootstrap(data=Cholesterol, boots=2);
run;
```

```
/*PRINT DATA */
proc print data=booted;
run;
ods rtf close;

/* RESIDUAL NORMALITY CHECKING*/
Data Booted;
Input choltotbayes hdl trig waist;
Cards;
207.54 73.00  91.00   96.50
190.94 45.00  123.00  95.00
193.40 66.00  81.00   85.00
202.99 42.00  139.00  81.00
207.54 73.00  91.00   96.50
190.94 45.00  123.00  95.00
203.47 39.00  151.00  94.00
202.99 42.00  139.00  81.00
170.81 43.00  98.00   90.00
137.86 42.00  62.00  104.00
203.47 39.00  151.00  94.00
264.48 56.00  207.00 113.00
190.94 45.00  123.00  95.00
193.40 66.00  81.00   85.00
264.48 56.00  207.00 113.00
137.86 42.00  62.00  104.00
193.40 66.00  81.00   85.00
303.12 70.00  222.00  91.50
204.00 57.00  126.00 116.50
220.11 92.00  71.00   97.50
178.03 55.00  85.00   92.00
175.88 45.00  100.00  88.00
207.54 73.00  91.00   96.50
170.81 43.00  98.00   90.00
202.99 42.00  139.00  81.00
137.86 42.00  62.00  104.00
156.15 46.00  76.00   98.00
220.11 92.00  71.00   97.50
193.40 66.00  81.00   85.00
229.48 71.00  122.00  91.50
204.00 57.00  126.00 116.50
137.86 42.00  62.00  104.00
202.99 42.00  139.00  81.00
203.47 39.00  151.00  94.00
202.99 42.00  139.00  81.00
178.03 55.00  85.00   92.00
203.47 39.00  151.00  94.00
190.12 47.00  118.00  95.00
203.47 39.00  151.00  94.00
189.41 49.00  118.00 106.50
190.12 47.00  118.00  95.00
189.41 49.00  118.00 106.50
137.86 42.00  62.00  104.00
175.88 45.00  100.00  88.00
137.86 42.00  62.00  104.00
224.26 32.00  193.00  95.50
202.99 42.00  139.00  81.00
204.00 57.00  126.00 116.50
264.48 56.00  207.00 113.00
144.42 47.00  58.00   97.00
204.00 57.00  126.00 116.50
202.99 42.00  139.00  81.00
203.47 39.00  151.00  94.00
137.86 42.00  62.00  104.00
202.99 42.00  139.00   81.00
207.54 73.00  91.00   96.50
175.88 45.00  100.00  88.00
```


10.12928/bamme.v2i1.3953

```
178.03 55.00  85.00   92.00
137.86 42.00  62.00  104.00
193.40 66.00  81.00   85.00
202.99 42.00  139.00  81.00
137.86 42.00  62.00  104.00
203.47 39.00  151.00  94.00
137.86 42.00  62.00  104.00
175.88 45.00  100.00  88.00
175.88 45.00  100.00  88.00
220.11 92.00  71.00   97.50
137.86 42.00  62.00  104.00
170.81 43.00  98.00   90.00
137.86 42.00  62.00  104.00
204.00 57.00  126.00 116.50
137.86 42.00  62.00  104.00
137.86 42.00  62.00  104.00
220.11 92.00  71.00   97.50
193.40 66.00  81.00   85.00
193.40 66.00  81.00   85.00
178.03 55.00  85.00   92.00
144.42 47.00  58.00   97.00
204.00 57.00  126.00 116.50
204.00 57.00  126.00 116.50
204.00 57.00  126.00 116.50
303.12 70.00  222.00  91.50
203.47 39.00  151.00  94.00
190.12 47.00  118.00  95.00
run;
;
Ods rtf file='abc.rtf'style=journal;
ods graphics on;
proc reg data=Booted plots=all;
model choltotbayes = hdl trig waist
          output   out=Residuals
p=y_hat
r=y_res;
run;
ods graphics off;

ods graphics on;
proc reg data=Booted plots=all;
model choltot = hdl trig waist/p ;
run;
ods graphics off;
ods rtf close;
run;

/* BAYESIAN REGRESSION MODEL*/
Data Booted;
Input choltotbayesian hdl trig waist;
Cards;
207.54 73.00  91.00   96.50
190.94 45.00  123.00  95.00
193.40 66.00  81.00   85.00
202.99 42.00  139.00  81.00
207.54 73.00  91.00   96.50
190.94 45.00  123.00  95.00
203.47 39.00  151.00  94.00
202.99 42.00  139.00  81.00
170.81 43.00  98.00   90.00
137.86 42.00  62.00  104.00
203.47 39.00  151.00  94.00
264.48 56.00  207.00 113.00
190.94 45.00  123.00  95.00
```

```
193.40 66.00  81.00   85.00
264.48 56.00  207.00 113.00
137.86 42.00  62.00  104.00
193.40 66.00  81.00   85.00
303.12 70.00  222.00  91.50
204.00 57.00  126.00 116.50
220.11 92.00  71.00   97.50
178.03 55.00  85.00   92.00
175.88 45.00  100.00  88.00
207.54 73.00  91.00   96.50
170.81 43.00  98.00   90.00
202.99 42.00  139.00  81.00
137.86 42.00  62.00  104.00
156.15 46.00  76.00   98.00
220.11 92.00  71.00   97.50
193.40 66.00  81.00   85.00
229.48 71.00  122.00  91.50
204.00 57.00  126.00 116.50
137.86 42.00  62.00  104.00
202.99 42.00  139.00  81.00
203.47 39.00  151.00  94.00
202.99 42.00  139.00  81.00
178.03 55.00  85.00   92.00
203.47 39.00  151.00  94.00
190.12 47.00  118.00  95.00
203.47 39.00  151.00  94.00
189.41 49.00  118.00 106.50
190.12 47.00  118.00  95.00
189.41 49.00  118.00 106.50
137.86 42.00  62.00  104.00
175.88 45.00  100.00  88.00
137.86 42.00  62.00  104.00
224.26 32.00  193.00  95.50
202.99 42.00  139.00  81.00
204.00 57.00  126.00 116.50
264.48 56.00  207.00 113.00
144.42 47.00  58.00   97.00
204.00 57.00  126.00 116.50
202.99 42.00  139.00  81.00
203.47 39.00  151.00  94.00
137.86 42.00  62.00  104.00
202.99 42.00  139.00  81.00
207.54 73.00  91.00   96.50
175.88 45.00  100.00  88.00
178.03 55.00  85.00   92.00
137.86 42.00  62.00  104.00
193.40 66.00  81.00   85.00
202.99 42.00  139.00  81.00
137.86 42.00  62.00  104.00
203.47 39.00  151.00  94.00
137.86 42.00  62.00  104.00
175.88 45.00  100.00  88.00
175.88 45.00  100.00  88.00
220.11 92.00  71.00   97.50
137.86 42.00  62.00  104.00
170.81 43.00  98.00   90.00
137.86 42.00  62.00  104.00
204.00 57.00  126.00 116.50
137.86 42.00  62.00  104.00
137.86 42.00  62.00  104.00
220.11 92.00  71.00   97.50
193.40 66.00  81.00   85.00
193.40 66.00  81.00   85.00
178.03 55.00  85.00   92.00
144.42 47.00  58.00   97.00
204.00 57.00  126.00 116.50
204.00 57.00  126.00 116.50
```


10.12928/bamme.v2i1.3953

```
204.00 57.00  126.00 116.50
303.12 70.00  222.00  91.50
203.47 39.00  151.00  94.00
190.12 47.00  118.00  95.00

;
run;

ods rtf file='abc.rtf'style=journal;
ods graphics on;
proc reg data=Booted plots=all;
run;

proc genmod data=Booted;

model choltotbayesian = hdl trig waist / dist=normal link=identity;

bayes seed=1 OutPost=Post diagnostics=all summary=all;;
run;
ods graphics off;

ods rtf close;
run;

/* BAYESIAN FUZZY REGRESSION*/

Title ‘Linear programming’;
data plant;
input  choltotbayesian hdl trig waist;
datalines;
207.54 73.00  91.00   96.50
190.94 45.00  123.00  95.00
193.40 66.00  81.00   85.00
202.99 42.00  139.00  81.00
207.54 73.00  91.00   96.50
190.94 45.00  123.00  95.00
203.47 39.00  151.00  94.00
202.99 42.00  139.00  81.00
170.81 43.00  98.00   90.00
137.86 42.00  62.00  104.00
203.47 39.00  151.00  94.00
264.48 56.00  207.00 113.00
190.94 45.00  123.00  95.00
193.40 66.00  81.00   85.00
264.48 56.00  207.00 113.00
137.86 42.00  62.00  104.00
193.40 66.00  81.00   85.00
303.12 70.00  222.00  91.50
204.00 57.00  126.00 116.50
220.11 92.00  71.00   97.50
178.03 55.00  85.00   92.00
175.88 45.00  100.00  88.00
207.54 73.00  91.00   96.50
170.81 43.00  98.00   90.00
202.99 42.00  139.00  81.00
137.86 42.00  62.00  104.00
156.15 46.00  76.00   98.00
220.11 92.00  71.00   97.50
193.40 66.00  81.00   85.00
229.48 71.00  122.00  91.50
204.00 57.00  126.00 116.50
137.86 42.00  62.00  104.00
202.99 42.00  139.00  81.00
203.47 39.00  151.00  94.00
```

```
202.99 42.00  139.00  81.00
178.03 55.00  85.00   92.00
203.47 39.00  151.00  94.00
190.12 47.00  118.00  95.00
203.47 39.00  151.00  94.00
189.41 49.00  118.00 106.50
190.12 47.00  118.00  95.00
189.41 49.00  118.00 106.50
137.86 42.00  62.00  104.00
175.88 45.00  100.00  88.00
137.86 42.00  62.00  104.00
224.26 32.00  193.00  95.50
202.99 42.00  139.00  81.00
204.00 57.00  126.00 116.50
264.48 56.00  207.00 113.00
144.42 47.00  58.00   97.00
204.00 57.00  126.00 116.50
202.99 42.00  139.00  81.00
203.47 39.00  151.00  94.00
137.86 42.00  62.00  104.00
202.99 42.00  139.00   81.00
207.54 73.00  91.00   96.50
175.88 45.00  100.00  88.00
178.03 55.00  85.00   92.00
137.86 42.00  62.00  104.00
193.40 66.00  81.00   85.00
202.99 42.00  139.00  81.00
137.86 42.00  62.00  104.00
203.47 39.00  151.00  94.00
137.86 42.00  62.00  104.00
175.88 45.00  100.00  88.00
175.88 45.00  100.00  88.00
220.11 92.00  71.00   97.50
137.86 42.00  62.00  104.00
170.81 43.00  98.00   90.00
137.86 42.00  62.00  104.00
204.00 57.00  126.00 116.50
137.86 42.00  62.00  104.00
137.86 42.00  62.00  104.00
220.11 92.00  71.00   97.50
193.40 66.00  81.00   85.00
193.40 66.00  81.00   85.00
178.03 55.00  85.00   92.00
144.42 47.00  58.00   97.00
204.00 57.00  126.00 116.50
204.00 57.00  126.00 116.50
204.00 57.00  126.00 116.50
303.12 70.00  222.00  91.50
203.47 39.00  151.00  94.00
190.12 47.00  118.00  95.00
;
run;
ods rtf file='result_ex1.rtf' ;

proc optmodel;
   set j= 1..84;
   number choltotbayesian{j}, hdl{j}, trig{j} waist{j};
   read data plant into [_n_]choltotbayesian hdl trig waist;

/*Print choltotbayesian  hdl  trig waist*/
print choltotbayesian  hdl  trig waist;

number n init 8;
/*Total number of Observations*/
/*Decision Variables*/
```

 10.12928/bamme.v2i1.3953

```
var aw{1..4}>=0;

/*Theses four variables are bounded*/

var ac{1..4};
/* These four variables are not bounded*/

/* Objective function*/
min z1=  aw[1] * n + sum{i in j} hdl[i] * aw[2] + sum{i in j} trig[i] *
aw[3]+ sum{i in j} waist[i] * aw[4];

/*Linear Constraints*/
con c{i in 1..n}:    ac[1]+hdl[i]*ac[2]+trig[i]*ac[3]+waist[i]*ac[4]-aw[1]-
hdl[i]*aw[2]- trig[i]*aw[3]- waist[i]*aw[4] <= choltotbayesian[i];

con c1{i in 1..n}: ac[1]+hdl[i]*ac[2]+trig[i]*ac[3]+waist[i]*ac[4] +aw[1]+
hdl[i]*aw[2]+trig[i]*aw[3]+waist[i]*aw[4] >= choltotbayesian[i];

expand;   /* This provides all equations */
solve;
print ac  aw;
quit;
ods  rtf  close;
```

Results and Discussion

Results from Bayesian multiple linear regression

The results from Bayesian multiple linear regression is presented in Table 2.

Table 2. Results from Bayesian multiple linear regression

Analysis of Maximum Likelihood Parameter Estimates				
Parameter	Estimate	Standard Error	Wald 95% Confidence Limits	
Intercept	62.5793	0.0028	62.5739	62.5847
Hdl (X_1)	1.4686	0.0000	1.4685	1.4686
Trig (X_2)	0.7511	0.0000	0.7511	0.7511
Waist (X_3)	-0.3170	0.0000	-0.3170	-0.3169

with Choltotbayesian (Y)

Multiple Bayesian Linear Regression (MBLR) is given as follows:

$$(Y) = 62.5793 + 1.4686\ (X_1) + 0.7511\ (X_2)\ -0.3170\,(X_3)$$

with

(X_1) is High Density Lipoprotein reading

(X_2) is a Triglycerides reading

(X_3) is Waist reading

Fitted Bayesian Multiple linear Regression with standard error is given as follows:

$$(Y) = 62.5793 + 1.4686\ (X_1) + 0.7511\ (X_2)$$

Std. Error (0.0028) (0.0000) (0.0000)

$-0.3170\,(X_3)$ ……………………… (2.1)

Std. error (0.0000)

Upper or lower limits of prediction interval are computed from the prediction equation (2.1) by taking the coefficient as their corresponding estimated values plus or minus standard error (See Table 3).

Upper limits

(Y) = 62.5821 + 1.4686 (X_1) + 0.7511 (X_2)-0.3170 (x_3).. (2.2)

Lower limits

(Y) = 62.5765 + 1.4686 (x_1) + 0.7511 (x_2)-0.3170 (x_3).. (2.3)

Table 3. Lower and upper limit with their width for Bayesian multiple linear regression

Lower limit	Upper limit	Width
207.544	207.550	.0056
190.934	190.939	.0056
193.398	193.404	.0056
202.984	202.989	.0056
207.544	207.550	.0056
190.934	190.939	.0056
203.470	203.476	.0056
⋮	⋮	⋮
303.117	303.123	.0056
203.470	203.476	.0056
190.116	190.121	.0056
	Average width	0.0056

Results from fitted model for Fuzzy regression is presented in Table 4.

Table 4. Value of center (AC) and radius (AW)

[1]	AC	AW
1	62.65633	0.0015848
2	1.46796	0.0000000
3	0.75085	0.0000000
4	-0.31716	0.0000000

Fitted model for fuzzy regression (FR) for Choltotbayesian (Y) =

<62.65633, 0.0015848> + < 1.46796, 0.0000000> Hdl + <0.75085, 0.0000000> Trig + <-0.31716, 0.0000000> Waist ....(2.4)

Upper or lower limits of prediction intervals are computed from the prediction equation (2.4) by taking the coefficient as their corresponding estimated values plus or minus standard error (See Table 5).

Upper limits

Y= <62.6547452> + < 1.46796, 0> (X_1) + <0.75085, 0> (X_2) +< -0.31716, 0> (X_3) ....(2.5)

Lower limits

Y= <62.6579148> + < 1.46796, 0> (X_1) +<0.75085, 0> (X_2) +<-0.31716, 0> (X_3) ....(2.6)

10.12928/bamme.v2i1.3953

Table 5. Lower and Upper Limit with Their Width for Fitted Model for Fuzzy Regression

Lower limit	Upper limit	Width
202.990	202.987	0.0032
207.540	207.537	.0032
190.940	190.937	.0032
203.474	203.470	.0032
202.990	202.987	.0032
170.819	170.816	.0032
137.880	137.877	.0032
⋮	⋮	⋮
303.084	303.081	0032
203.474	203.470	.0032
190.122	190.119	.0032
Average width 0.003170		

The width of prediction intervals concerning the Bayesian multiple linear regression model and the Bayesian fuzzy regression model corresponding to each set of observed explanatory variables is computed in SPSS and the results are reported in Table 5. From this table, the average width for the former was found to be 0.005600, while that of the latter was only 0.003170, thereby indicating the superiority of fuzzy regression methodology.

Conclusion

This paper presents an algorithm and illustrated the procedure of modeling by using modified Bayesian linear regression through SAS language. Our aim is to share the algorithm and also provide the researcher with an alternative programming that suitable for a small sample size. This proposed method can be applied to small sample size data, especially when limited data is obtained, for example in public health.

Acknowledgement

The authors would like to express their gratitude to Universiti Sains Malaysia (USM) for providing International Book Grant (Grant No.304/PPSG/6150179/T150), School of Dental Sciences).

References

Diem Ngo, T. H., & La Puente, C. A. (2012). The Steps to Follow in a Multiple Regression Analysis. SAS Global Forum 2012. *Statistics and Data Analysis*, Paper *333-2012*, 1-12.

Cassel, D. L. (2010). Bootstrap Mania: Re sampling the SAS. SAS Global Forum 2010: Statistics and Data Analysis. Paper 268-279. *Proceedings of the SAS Global Forum 2010 Conference.* Cary (NC): SAS Institute Inc.

Jung, B. C., Jhun, M., & Lee, J. W. (2005). Bootstrap Tests for Overdispersion in a Zero-Inflated Poisson Regression Model. *Biometrics, 61*, 626-629.

Kacprzyk J., & Fedrizzi M. (1992) *Fuzzy Regression Analysis*, Warsaw: Omnitech Press.

Bradley, E., & Tibshirani, R. J. (1993). *An Introduction to the Bootstrap*. New York, NY: Chapman and Hall.

Higgins, G. E. (2005). Statistical Significance Testing: The Bootstrapping Method and an Application to Self-Control Theory. *The Southwest Journal of Criminal Justice*, *2(1)*, 54-76.

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2013). *Bayesian Data*

Analysis, third edition. USA: CRC Press.

Gelman, A., & Hill, J. (2006). *Data Analysis Using Regression and Multilevel/Hierarchical Models*. Cambridge: Cambridge University Press.

Osborne, J.W. (2010). Improving your data transformations?: Applying the Box-Cox transformation. Practical Assessment, *Research & Evaluation*, *15(12),* 1-9.