Bulletin of Applied Mathematics and Mathematics Education
Volume 5, Number 2, October 2025 pp. 121-134
https://dx.doi.org/10.12928/bamme.v5i2.14795
e-ISSN 2776-1029, p-ISSN 2776-1002

Mathematical model of social media addiction: An optimal control approach

Ratna Widayati[1*], Intrada Reviladi[2], Nur Afandi[1], Ramya Rachmawati[1]

[1]Universitas Bengkulu, Jl. W.R Supratman Kandang Limun, Bengkulu 38112, Indonesia
[2]Universitas Jenderal Soedirman, Jl. Prof. Dr. HR. Boenjamin No. 993, Banyumas, Jawa Tengah 53112, Indonesia

*Corresponding E-mail: rwidayati@unib.ac.id

ARTICLE INFO

Article History
Received 24 October 2025
Revised 14 January 2025
Accepted 14 January 2026

Keywords
Social media addiction
Mathematical model
Pontryagin's
Minimum Principle
Optimal control

How to cite this article:
Widayati, R., Reviladi, I., Afandi, N., & Rachmawati, R. (2026). Mathematical model of social media addiction: An optimal control approach. *Bulletin of Applied Mathematics and Mathematics Education*, *5*(2), 121-134.

ABSTRACT

In this study, we develop a deterministic SEARQS compartmental mathematical model to analyze the dynamics of social media addiction and evaluate the effectiveness of optimal control strategies. The model describes the transitions of individuals among susceptible (S), exposed (E), addicted (A), recovered (R), and quarantined (Q) classes. Two time-dependent control variables are incorporated into the model: the first control, u_1 represents preventive awareness efforts that reduce the transitions from susceptible individuals to the exposed and quarantined classes through education and public awareness campaigns, while the second control, u_2 represents treatment interventions that enhance recovery by increasing the transitions from the exposed and addicted classes to the recovered class. An optimal control problem is formulated and analyzed using Pontryagin's Minimum Principle to derive the necessary optimality conditions. Numerical simulations are performed using the forward–backward sweep method to assess the impact of the proposed strategies. Simulation results indicate that, compared to the uncontrolled scenario, the combined awareness and treatment controls reduce the peak number of exposed individuals by approximately 14% and achieve an estimated 50% reduction in the peak prevalence of addicted individuals. These findings highlight the effectiveness of integrated intervention strategies and support their implementation as practical policy measures to mitigate the adverse effects of social media addiction on public health and societal well-being.

Introduction

Interpersonal communication is a fundamental human need, and the rapid development of internet-based social media platforms has profoundly transformed how individuals establish and maintain social connections. Despite their benefits in facilitating instant communication and information sharing, excessive and uncontrolled use of social media has been increasingly associated with behavioral addiction. Social media addiction is commonly characterized by compulsive and prolonged engagement that interferes with daily activities, responsibilities, and social functioning, potentially leading to diminished psychological well-being, strained interpersonal relationships, and reduced academic or occupational performance (Hou et al. 2019). A growing body of empirical research has further linked excessive social media use to adverse mental health outcomes, including increased anxiety, loneliness, and suicidal ideation (Pellegrino, Stasi, and Bhatiasevi 2022).

As of 2022, approximately 4.62 billion individuals out of a global population of 7.91 billion were active social media users worldwide. On average, individuals spent nearly 6 hours and 58 minutes online per day, with about 2 hours and 27 minutes devoted specifically to social media usage. This widespread and prolonged engagement underscores the central role of social media in everyday life, while simultaneously raising concerns about excessive exposure and the emergence of addictive behavioral patterns. Although these aggregated statistics are not directly used to estimate the model parameters in this study, they provide important contextual justification for the assumed intensity of exposure and interaction. In particular, prolonged daily usage suggests frequent opportunities for behavioral influence, supporting the choice of moderate-to-high exposure and relapse rates. Future studies may utilize platform-specific usage data or longitudinal surveys to calibrate transmission, quitting, and recovery parameters more precisely.

Social media addiction may propagate through mechanisms of behavioral and social contagion rather than biological transmission. Individuals exhibiting addictive usage patterns can influence susceptible users through repeated exposure to highly engaging content, such as frequent posts, algorithm-driven recommendations, online challenges, influencer promotions, and implicit social pressure to remain active. These mechanisms contribute to normalization of excessive usage and intensify fear of missing out (FOMO), particularly among adolescents and young adults who are highly responsive to peer validation and social belonging (Przybylski et al. 2013). As a result, casual engagement may gradually evolve into compulsive use, allowing addictive behaviors to spread across social networks in a manner analogous to a behavioral epidemic (Bather et al. 2012). To capture this population-level influence, the present study adopts the mass action principle as a first-order approximation of interaction between susceptible and addicted individuals. While this assumption simplifies heterogeneous interaction patterns and algorithmic biases inherent in real social networks, it provides a tractable framework for analyzing large-scale addiction dynamics. The limitations of this approach motivate future extensions incorporating network-based or frequency-dependent transmission mechanisms.

Mathematical modeling has been widely applied to study the spread and control of addictive behaviors, including substance abuse and behavioral addictions such as online gaming and social media use (Alemneh and Alemu 2021). Previous studies have examined alcohol consumption dynamics influenced by media exposure (Ma, Huo, and Meng 2015; Sharma and Samanta 2013), the role of media in disease transmission (Cui, Sun, and Zhu 2008), and treatment-based interventions for alcohol dependence (Khajji et al. 2020). In the context of behavioral addiction, (Guo and Li 2020) proposed a five-compartment model for online gaming addiction, while (Ishaku et al. 2018) investigated the impact of social media use on academic performance by categorizing users based on activity levels. Compartmental models for social media addiction have also been developed in (Alemneh 2020; Al Addawiyah and Fuad 2023), where populations were divided into multiple user classes. However, these models often assume permanent recovery, include addiction-induced mortality, or incorporate transition probabilities for low-frequency users, which may not accurately reflect the cyclical nature of social media engagement.

Recent research has applied mathematical modeling to understand and control social media addiction. Fractional-order models have been shown to capture memory effects and long-term dependence in addictive behavior (Shutaywi, Rehman, and Shah 2023), while equilibrium and

 10.12928/bamme.v5i2.14795

stability analyses have been facilitated through computational approaches (KOCABIYIK 2025). Epidemiological frameworks such as SIR-type models indicate that social media addiction can persist when the reproduction threshold exceeds unity (Side, Sanusi, and Rustan 2020). Numerical studies further emphasize the role of high-order schemes in accurately describing addiction dynamics (Fatahillah A 2025). In addition, SEI_1I_2R models applied to TikTok addiction reveal that heterogeneous exposure and addiction severity significantly affect prevalence outcomes(Abi et al. 2023; Abi et al. 2023).

Several mathematical modeling studies have been conducted to analyze the dynamics and control of social media addiction. (Pagga et al. 2015) investigated a compartmental model and demonstrated that the stability of the addiction-free equilibrium is strongly influenced by exposure and recovery rates. Using a simpler SIR framework (Lestari et al. 2025) showed that increasing the recovery rate significantly reduces the number of addicted individuals among university students. (Romlah, Thahiruddin, and Sarifah 2025) further explored the numerical behavior of a TikTok addiction model, highlighting the sensitivity of long-term addiction dynamics to key transition parameters. Extending the scope to mental health outcomes, (Ali et al. 2024) developed a coupled mathematical model linking social media addiction and depression, showing that effective control of addictive behavior can indirectly mitigate adverse psychological effects. From an optimal control perspective, (Kamal, Ahmed, and Sarkar 2025) demonstrated that time-dependent intervention strategies are capable of reducing the basic reproduction number and achieving cost-effective addiction control. Similarly, (Juhari and Alisah 2024) analyzed an optimal control model and concluded that a combination of preventive awareness and treatment-based interventions yields the most effective reduction in addiction prevalence. Collectively, these studies underscore the importance of recovery mechanisms, parameter sensitivity, and adaptive control strategies in understanding and mitigating social media addiction.

In contrast to existing approaches, the present study formulates a SEARQ compartmental model that explicitly incorporates a quitting (Q) class to represent temporary disengagement from social media, together with relapse pathways that allow individuals in the recovered or quitting states to return to exposure or addiction. This structure reflects empirical evidence indicating that social media addiction is typically characterized by recurrent cycles of abstinence and relapse rather than permanent cessation. Mortality due to addiction is excluded, consistent with the non-fatal nature of social media overuse, and transitions involving low-frequency users are omitted to concentrate on the dominant addiction dynamics. These modeling choices yield a more realistic framework for capturing long-term behavioral patterns and for evaluating the effectiveness of intervention strategies.

Several studies have previously investigated mathematical models of online behavioral addictions.(Karimah 2022) analyzed an online gaming addiction model by examining equilibrium stability and implementing optimal control to reduce addiction prevalence. (Firdaus and Krisnawan 2023) developed a TikTok addiction model based on survey data using an SA_1A_2R structure, predicting that severe addiction would remain dominant over mild addiction, although both decline over time. In contrast to these studies, the present model explicitly incorporates quitting and relapse mechanisms within an optimal control framework, enabling a more comprehensive assessment of prevention and treatment policies over extended time horizons.

Building upon the work of (Widayati and Reviladi 2023), which analyzed the local stability of the disease-free equilibrium for the same SEARQ model without control measures, this study integrates optimal control strategies representing preventive awareness and treatment interventions. The proposed framework enables the assessment of combined strategies aimed at reducing exposure, promoting recovery, and mitigating relapse, thereby offering actionable insights for policymakers and stakeholders seeking to address the societal impacts of social media addiction.

Method

Model Formulation

In this section, we develop a deterministic mathematical model to describe the dynamics of social media addiction, based on the following assumptions: the phenomenon occurs within a closed population; factors such as sex, race, and social status do not influence the likelihood of becoming addicted to social media; and individuals within the population are assumed to interact homogeneously, meaning each person has the same level and type of interaction with others.
The model is built upon several key assumptions. First, the natural birth rate is assumed to be equal to the natural death rate, ensuring a constant total population over time. Second, deaths resulting directly from social media addiction are not considered in this model. Third, individuals who have either never used social media or who have ceased using it permanently may still return to a susceptible state, meaning they can potentially become users again. Lastly, all population compartments are assumed to experience the same natural death rate.

In constructing this model, the population is assumed to be divided into five distinct compartments. The susceptible individuals (s) are those who are vulnerable to developing social media addiction. The exposed individuals (e) are those who use social media but have not yet developed signs of addiction. The addicted individuals (a) represent those who are currently addicted to social media. The recovered individuals (r) are those who have overcome their addiction. Lastly, the quit individuals (q) are those who have completely stopped using social media.

Newborn individuals enter the susceptible population at a natural birth rate denoted by μ. Susceptible individuals interact with those who are addicted to social media at an interaction rate β. This interaction influences susceptible individuals to begin using social media, transitioning them into the exposed class i.e. individuals who use social media but are not yet addicted. Exposed individuals may develop an addiction and move into the addicted class at a rate δ while some exposed individuals recover through treatment at a rate α. Addicted individuals may transition to the recovered class at a rate ε as a result of education and/or rehabilitation. Recovered individuals may become susceptible again to social media addiction at a rate γ while some may completely cease using social media at a rate η. Susceptible individuals may also quit social media at a rate κ whereas those in the quit class can return to the susceptible class at a rate χ. All population compartments are assumed to experience the same natural death rate μ.

In this section, we aim to minimize the prevalence of social media addiction through the introduction of control variables. The first control variable $u_1(t)$ represents preventive efforts designed to reduce contact between susceptible individuals and those who are addicted. These efforts may include public awareness campaigns and educational programs that emphasize the negative consequences of excessive social media use. The second control variable $u_2(t)$ is implemented to manage addicted individuals by providing appropriate treatment interventions, as described in the introduction, to support their recovery. The following diagram illustrates the transition dynamics representing the described control framework.

 10.12928/bamme.v5i2.14795

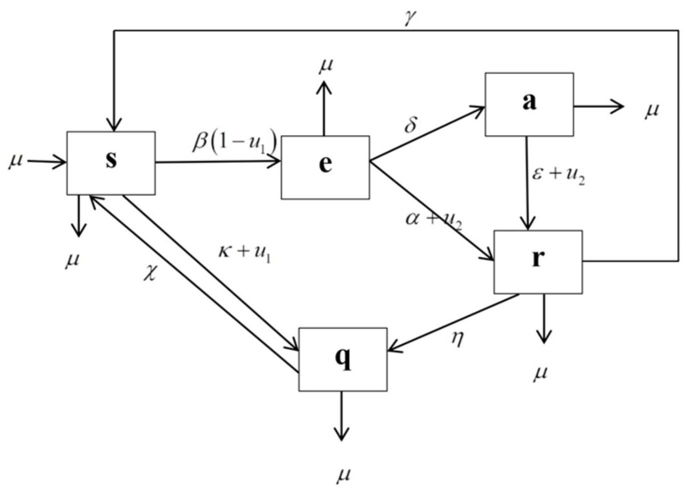

Figure 1. Compartmental diagram of social media addiction dynamics with control measures.

Taking into account the considerations outlined above and the flow diagram presented in Figure 1, the dynamics of social media addiction within the human population can be described by the following system of nonlinear differential equations:

$$\begin{aligned}
\frac{ds}{dt} &= \mu-(\mu+\kappa+u_1)s+\gamma r-(1-u_1)\beta sa+\chi q \\
\frac{de}{dt} &= (1-u_1)\beta sa-(\mu+\delta+\alpha+u_2)e \\
\frac{da}{dt} &= \delta e-(\mu+\varepsilon+u_2)a \\
\frac{dr}{dt} &= (\alpha+u_2)e+(\varepsilon+u_2)a-(\mu+\gamma+\eta)r \\
\frac{dq}{dt} &= \eta r+\kappa s+u_1 s-(\mu+\chi)q.
\end{aligned} \tag{1}$$

Optimal Control Analysis

To facilitate the study of the optimal control problem, we define the control set U as follows:

$$U=\{(u_1(t),u_2(t)): 0\le u_1(t)\le 1, 0\le u_2(t)\le 1, 0\le t\le t_f\}.$$

The integration of control measures into the model aims to determine the optimal intensity of intervention strategies that effectively reduce both the spread of social media addiction and the associated implementation costs. The control variables u_1 and u_2 are optimized subject to the constraints imposed by the system of differential Equations (1), with the objective functional defined as follows:

$$J(u_1,u_2)=\int_0^{t_f}\left(c_1 e+c_2 a+\frac{1}{2}c_3 u_1^2+\frac{1}{2}c_4 u_2^2\right)dt. \tag{2}$$

Here, t_f represents the final time, while c_1 and c_2 serve as weighting parameters associated with the Exposed and Addicted compartments, respectively. Similarly, c_3 and c_4 represent the weighting coefficients corresponding to each control measure. Following the formulation of the optimal control problem, the existence of optimal control variables is established. Thereafter, the Pontryagin Minimum Principle is applied to characterize the optimal controls and derive the corresponding optimality conditions for the model.

We examine the existence of an optimal control for the proposed problem by applying the theoretical framework developed by Fleming and Rishel (Bather, Fleming, and Rishel 1976).

Theorem 1. There exists an optimal control pair $u^* = (u_1^*, u_2^*) \in U$ such that

$$J(u_1^*, u_2^*) = min\{J(u_1, u_2) | u_1(t), u_2(t) \in U\}$$

where $U = \{(u_1(t), u_2(t)): 0 \le u_1(t) \le 1, 0 \le u_2(t) \le 1, 0 \le t \le t_f\}$ is a closed admissible control set. This optimization is subject to the control system defined in Equations (1), along with the specified initial conditions.

Proof of Theorem 1.

To establish the existence of an optimal control, we follow the standard framework in optimal control theory as described in (Bather et al. 1976). The admissible control set U is nonempty, convex, closed, and bounded in $L^\infty(0, t_f)$. For any admissible control $(u_1, u_2) \in U$, the corresponding state system admits a unique solution, and all state variables remain nonnegative and bounded for all $t \in [0, t_f]$. Moreover, the right-hand side of the state system depends linearly on the control variables and is bounded by a linear function of the state and control variables. The integrand of the objective functional is convex with respect to the control variables. In addition, there exist positive constants c_3 and c_4 and an exponent $k > 1$ such that the growth condition

$$L(x, u) \ge c_3 \|u\|^k - c_4$$

is satisfied for $k = 2$.

Therefore, by standard existence results for optimal control problems (see Fleming and Rishel [20]), there exists an optimal control pair $u^* = (u_1^*, u_2^*) \in U$ that minimizes the objective function. ■

The following section is devoted to the application of the Pontryagin Minimum Principle in order to identify the optimal control that satisfies the necessary conditions. This principle is employed to reformulate Equations (1) and (2) into a pointwise Hamiltonian minimization problem, with the objective of determining the optimal control functions $u_1(t)$ and $u_2(t)$. The corresponding Hamiltonian function is defined as follows:

$$\begin{aligned} H &= c_1 e + c_2 a + \frac{1}{2} c_3 u_1^2 + \frac{1}{2} c_4 u_2^2 + \lambda_1 \left(\mu - (\mu + \kappa + u_1) s + \gamma r - (1 - u_1) \beta s a + \chi q \right) \\ &+ \lambda_2 \left((1 - u_1) \beta s a - (\mu + \delta + \alpha + u_2) e \right) + \lambda_3 \left(\delta e - (\mu + \varepsilon + u_2) a \right) \\ &+ \lambda_4 \left(\alpha e + u_2 e + \varepsilon a + u_2 a - (\mu + \gamma + \eta) r \right) + \lambda_5 \left(\eta r + \kappa s + u_1 s - (\mu + \chi) q \right), \end{aligned}$$

where λ_i for $i = 1, 2, \ldots, 5$ denote the adjoint variables to be determined.

Theorem 2. *Let $u^* = (u_1^*, u_2^*)$ be an optimal control in the control space U, along with the solution $s^*(t), e^*(t), a^*(t), r^*(t), q^*(t)$ for the associated state system defined by Eqs (1) and (2). There exist adjoint variables λ_i for $i = 1, 2, \ldots, 5$ that satisfy the equation below*

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial s}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial e}$$

$$\dot{\lambda}_3 = -\frac{\partial H}{\partial a}$$

$$\dot{\lambda}_4 = -\frac{\partial H}{\partial r}$$

 10.12928/bamme.v5i2.14795

$$\dot{\lambda}_5 = -\frac{\partial H}{\partial q}$$

with transversality conditions $\lambda_i(t_f)$ *for* $i = 1,2,\dots,5$ *and control set* $u^* = (u_1^*, u_2^*)$ *characterized by*

$$u_1^* = \max\left\{0, \min\left\{1, \frac{\lambda_2 \beta s^* a^* - \lambda_5 s - \lambda_1(\beta s^* a^* - s^*)}{c_3}\right\}\right\}$$

and

$$u_2^* = \max\left\{0, \min\left\{1, \frac{e^*\lambda_2 + \lambda_3 a^* - \lambda_4 e^* - \lambda_4 a^*}{c_4}\right\}\right\}.$$

Proof of Theorem 2. According to Pontryagin's Minimum Principle, differentiating the Hamiltonian yields the adjoint system, which can be expressed as follows:

$$\begin{aligned}
\dot{\lambda}_1 &= -\frac{\partial \mathrm{H}}{\partial s} = -\lambda_1\left(-\mu-\kappa-u_1-(1-u_1)\beta a\right)-\lambda_2(1-u_1)\beta a-\lambda_5(\kappa+u_1)\\
\dot{\lambda}_2 &= -\frac{\partial \mathrm{H}}{\partial e} = -c_1-\lambda_2(-\mu-\delta-\alpha-u_2)-\lambda_3\delta-\lambda_4(\alpha+u_2)\\
\dot{\lambda}_3 &= -\frac{\partial \mathrm{H}}{\partial a} = -c_2+\lambda_1(1-u_1)\beta s-\lambda_2(1-u_1)\beta s-\lambda_3(-\mu-\varepsilon-u_2)-\lambda_4 u_2\\
\dot{\lambda}_4 &= -\frac{\partial \mathrm{H}}{\partial r} = -\lambda_1\gamma-\lambda_4(-\mu-\gamma-\eta)-\lambda_5\eta\\
\dot{\lambda}_5 &= -\frac{\partial \mathrm{H}}{\partial q} = -\lambda_1\chi-\lambda_5(-\mu-\chi)
\end{aligned}$$

with transversality conditions $\lambda_i(t_f)$ for $i = 1,2,\dots,5$. The control set $u^* = (u_1^*, u_2^*)$ satisfies the condition

$$\frac{\partial \mathrm{H}}{\partial u_1} = 0$$

$$u_1 = \frac{\lambda_2 \beta s^* a^* - \lambda_5 s^* - \lambda_1(\beta s^* a^* - s^*)}{c_3},$$

and

$$\frac{\partial \mathrm{H}}{\partial u_2} = 0$$

$$u_2 = \frac{e^*\lambda_2 + \lambda_3 a^* - \lambda_4 e^* - \lambda_4 a^*}{c_4}.$$

By applying standard control methods and considering the constraints on the control values within the control set U, the forms of u_1 and u_2 can be readily determined. This is accomplished by following established procedures in control theory and accounting for the upper and lower bounds imposed on the admissible controls, as outlined below

$$u_1^* = \max\left\{0, \min\left\{1, \frac{\lambda_2 \beta s^* a^* - \lambda_5 s - \lambda_1(\beta s^* a^* - s^*)}{c_3}\right\}\right\}$$

and

$$u_2^* = \max\left\{0, \min\left\{1, \frac{e^* \lambda_2 + \lambda_3 a^* - \lambda_4 e^* - \lambda_4 a^*}{c_4}\right\}\right\}. \blacksquare$$

The optimality system is composed of the optimal control system (state equations) and the adjoint system, combined with the set of control functions characterized by the initial and transversality conditions:

$$\begin{cases} \frac{\partial H}{\partial \lambda_1} = \mu - (\mu + \kappa + u_1)s + \gamma r - (1-u_1)\beta sa + \chi q \\ \frac{\partial H}{\partial \lambda_2} = (1-u_1)\beta sa - (\mu + \delta + \alpha + u_2)e \\ \frac{\partial H}{\partial \lambda_3} = \delta e - (\mu + \varepsilon + u_2)a \\ \frac{\partial H}{\partial \lambda_4} = \alpha e + u_2 e + \varepsilon a + u_2 a - (\mu + \gamma + \eta)r \\ \frac{\partial H}{\partial \lambda_5} = \eta r + \kappa s + u_1 s - (\mu + \chi)q \\ -\frac{\partial H}{\partial s} = -\lambda_1(-\mu - \kappa - u_1 - (1-u_1)\beta a) - \lambda_2(1-u_1)\beta a - \lambda_5(\kappa + u_1) \\ -\frac{\partial H}{\partial e} = -c_1 - \lambda_2(-\mu - \delta - \alpha - u_2) - \lambda_3\delta - \lambda_4(\alpha + u_2) \\ -\frac{\partial H}{\partial a} = -c_2 + \lambda_1(1-u_1)\beta s - \lambda_2(1-u_1)\beta s - \lambda_3(-\mu - \varepsilon - u_2) - \lambda_4 u_2 \\ -\frac{\partial H}{\partial r} = -\lambda_1\gamma - \lambda_4(-\mu - \gamma - \eta) - \lambda_5\eta \\ -\frac{\partial H}{\partial q} = -\lambda_1\chi - \lambda_5(-\mu - \chi) \\ u_1^* = \max\left\{0, \min\left\{1, \frac{\lambda_2\beta s^* a^* - \lambda_5 s - \lambda_1(\beta s^* a^* - s^*)}{c_3}\right\}\right\} \\ u_2^* = \max\left\{0, \min\left\{1, \frac{e^* \lambda_2 + \lambda_3 a^* - \lambda_4 e^* - \lambda_4 a^*}{c_4}\right\}\right\} \\ \lambda_i(t_f) = 0, s(0) = s_0, e(0) = e_0, a(0) = a_0, r(0) = r_0, q(0) = q_0. \end{cases} \tag{3}$$

Results and Discussion

Numerical Simulation

To carry out the numerical simulations, we employ the parameter values presented in the table below. These parameters are essential for solving the optimality system and for analyzing the dynamic behavior of the model under specified conditions. Numerical simulation plays a vital role in the study of epidemiological problems, particularly when analytical solutions to complex nonlinear systems are difficult or impossible to derive. Through simulation, we gain a deeper understanding of the progression of social media addiction, assess the effectiveness of various control strategies, and provide valuable insights to support optimal decision-making in public health interventions.

 10.12928/bamme.v5i2.14795

Table 1. Description of parameters of the social media addiction model (1).

Parameter	Description	Value	Source
μ	Natural death rate/ natural birth rate	0.25	(Alemneh and Alemu 2021)
β	Interaction rate/ transmission rate	0.6	(Wang et al. 2014)
δ	The rate at which exposed individuals become addicted	0.175	(Guo and Li 2020)
α	The rate at which exposed individuals recover through treatment	0.075	(Guo and Li 2020)
ε	Addicted individuals that join recovered class due to the treatment	0.7	(Huo and Wang 2014)
η	The rate at which recovered individuals permanently stop using social media	0.26	(Guo and Li 2020)
γ	The rate at which recovered individuals become susceptible again	0.14	(Huo and Wang 2014)
κ	The rate at which susceptible individuals quit social media	0.01	(Alemneh and Alemu 2021)
χ	The rate at which individuals in the quit class return to being susceptible	0.2	Assumed

In this section, we perform a numerical simulation of the optimality system using MATLAB. To simulate the optimality system presented in equation (6), we implemented the model computationally using the forward-backward sweep method. This method involves solving the state equations forward in time and the adjoint equations backward in time, iteratively updating the control variables until convergence is achieved. For the simulation, we used the parameter values listed in Table 1 and the following initial conditions: $s(0) = 1000$, $e(0) = 10$, $a(0) = 50$, $r(0) = 0$ and $q(0) = 100$. Additionally, the weight constants were set as follows: $c_1 = 1$, $c_2 = 2$, $c_3 = 10$ and $c_4 = 10$ (Alemneh 2020).

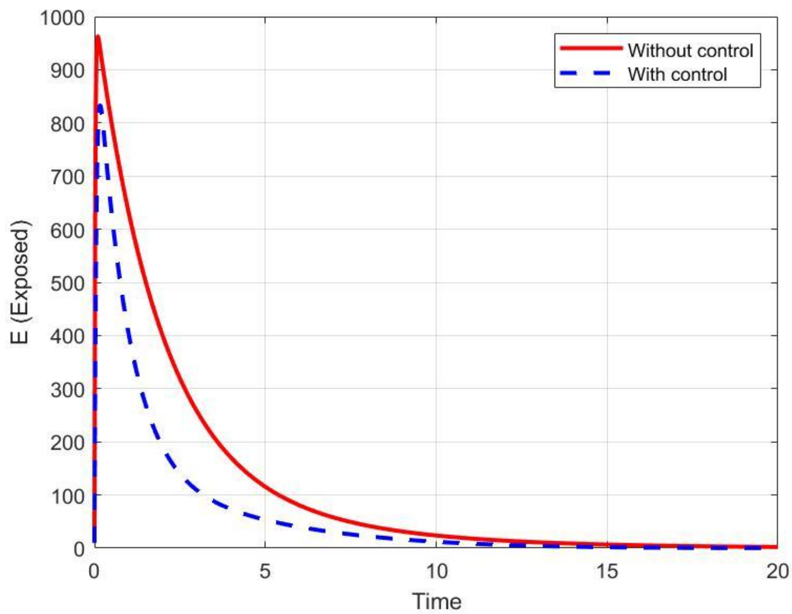

Figure 2. The impact of optimal control strategies on the exposed compartment.

Figure 2 shows the decline of the exposed population over time, comparing scenarios with and without control measures. Without control (red line), the decline is slower, indicating prolonged exposure. In contrast, with control (blue dashed line), the number of exposed individuals drops rapidly. This highlights the effectiveness of control measures in reducing exposure more quickly and efficiently.

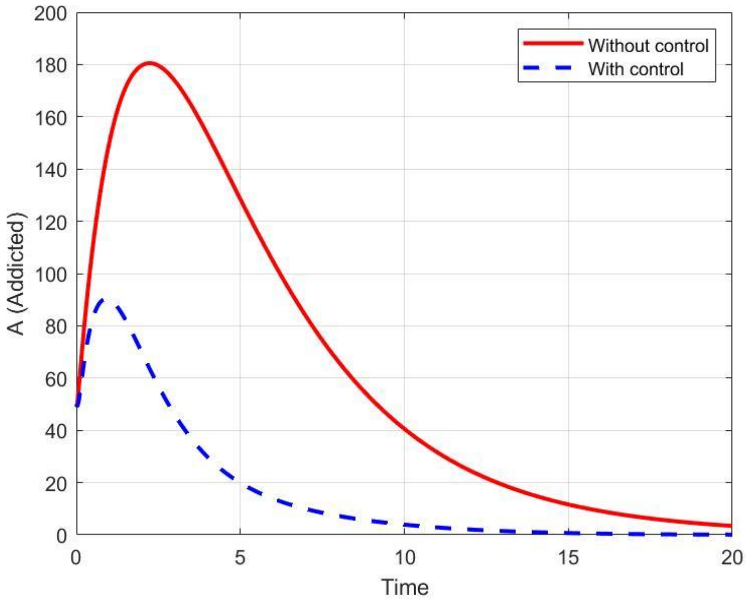

Figure 3. The impact of optimal control strategies on the addicted compartment.

Figure 3 illustrates the number of addicted individuals over time under two scenarios: with and without control measures. Without control (red line), the addicted population rises to a higher peak and declines slowly. In contrast, with control (blue dashed line), the peak is lower and the decline occurs more rapidly. This indicates that control interventions significantly reduce both the severity and duration of addiction in the population.

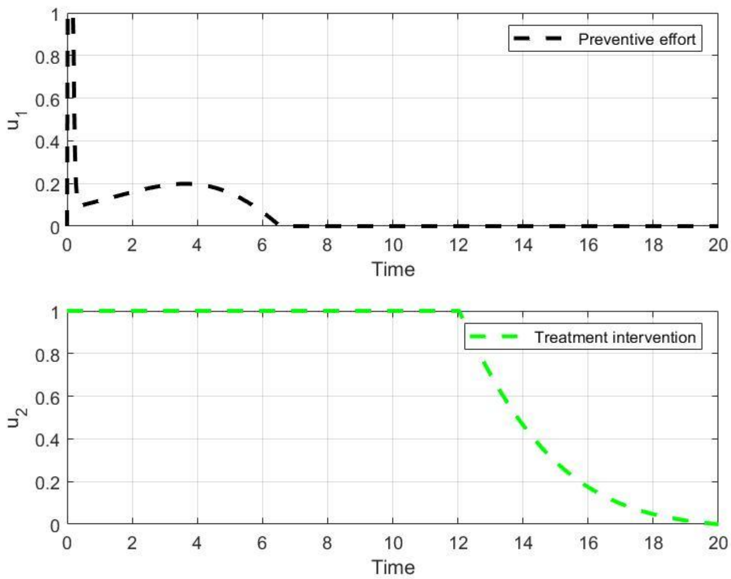

Figure 4. Optimal control profile.

 10.12928/bamme.v5i2.14795

Figure 4 presents the optimal time profiles of the control strategies: preventive effort (u_1) and treatment intervention (u_2). The preventive effort peaks briefly at the beginning and then decreases to zero around $t = 7$, indicating its importance in the early phase. Meanwhile, the treatment intervention remains at its maximum level until around $t = 13$, after which it gradually declines. This suggests that prevention is most effective early on, while treatment plays a sustained role before being gradually reduced.

Conclusion

This study developed and analyzed a SEARQS compartmental model with optimal control to investigate the dynamics of social media addiction and the effectiveness of preventive and treatment-based interventions. By incorporating awareness campaigns and treatment programs as time-dependent control variables, the proposed framework provides both qualitative and quantitative insights into addiction mitigation strategies.

A key theoretical finding of this work is that the optimal control strategy successfully maintains the basic reproduction number R_0 below unity. This result guarantees, from a mathematical perspective, that the prevalence of social media addiction declines over time under the combined intervention strategy. In contrast, scenarios without control or with isolated interventions fail to consistently keep $R_0 < 1$ allowing addiction to persist within the population. This highlights the critical role of integrated and well-coordinated control measures.

Numerical simulations further demonstrate that the optimal controls are inherently time-dependent, with stronger interventions required during the early stages of exposure and addiction, followed by gradual relaxation as the system approaches stability. These findings confirm that static or constant-intensity policies are suboptimal and may lead to unnecessary costs or reduced effectiveness. Instead, adaptive control strategies achieve superior outcomes while balancing intervention costs and public health benefits.

From a policy perspective, the results strongly recommend the simultaneous implementation of preventive awareness campaigns and targeted treatment programs. Public education initiatives should focus on reducing initial exposure, while rehabilitation and support services should be dynamically adjusted to facilitate recovery and prevent relapse. Overall, this study underscores that adaptive, time-dependent intervention strategies are essential for cost-effective and sustainable control of social media addiction, offering valuable guidance for policymakers, educators, and public health stakeholders.

References

Abi, Maria M., Elinora N. Bano, Leonardus F. Obe, and Fried M. A. Blegur. 2023. "Pemodelan Matematika Dan Simulasi Kecanduan Media Sosial." 5(1):43–55.

Al Addawiyah, Amartya Fierzi, and Yusuf Fuad. 2023. "Model Dinamik SEARQ Dan Penerapan Kontrol Optimal Pada Permasalahan Kecanduan Media Sosial." MATHunesa: Jurnal Ilmiah Matematika 11(1):67–81. doi: 10.26740/mathunesa.v11n1.p67-81.

Alemneh, Haileyesus Tessema. 2020. "Mathematical Modeling, Analysis, and Optimal Control of Corruption Dynamics." Journal of Applied Mathematics 2020. doi: 10.1155/2020/5109841.

Alemneh, Haileyesus Tessema, and Negesse Yizengaw Alemu. 2021. "Mathematical Modeling with Optimal Control Analysis of Social Media Addiction." Infectious Disease Modelling 6:405–19. doi: 10.1016/j.idm.2021.01.011.

Ali, Abu Safyan, Shumaila Javeed Id, Zeshan Faiz, and Dumitru Baleanu. 2024. Mathematical Modelling , Analysis and Numerical Simulation of Social Media Addiction and Depression.

Bather, J. A., W. H. Fleming, and R. W. Rishel. 1976. Deterministic and Stochastic Optimal Control. Vol. 139.

Cui, Jingan, Yonghong Sun, and Huaiping Zhu. 2008. "The Impact of Media on the Control of Infectious Diseases." Journal of Dynamics and Differential Equations 20(1):31–53. doi: 10.1007/s10884-007-9075-0.

Fatahillah, A., Prihandini, R. M., Hidayatullah, A., Setiawani, S., & Adawiyah, R. (2025). Numerical simulation of a social media addiction dynamic model using the 15th-order Runge–Kutta method. Jurnal Matematika UNAND, 14(4), 400–410.Firdaus, K. A., and K. P. Krisnawan. 2023. "Model Matematika Dari Dinamika Kecanduan Media Sosial Tiktok Pada Mahasiswa Universitas Negeri Yogyakarta Angkatan 2019." Jurnal Sains Dasar 12(2):101–7.

Guo, Youming, and Tingting Li. 2020. "Optimal Control and Stability Analysis of an Online Game Addiction Model with Two Stages." Mathematical Methods in the Applied Sciences 43(7):4391–4408. doi: 10.1002/mma.6200.

Hou, Y., D. Xiong, T. Jiang, L. Song, and ... 2019. "Social Media Addiction: Its Impact, Mediation, and Intervention." ... : Journal of Psychosocial ....

Huo, Hai Feng, and Qian Wang. 2014. "Modelling the Influence of Awareness Programs by Media on the Drinking Dynamics." Abstract and Applied Analysis 2014. doi: 10.1155/2014/938080.

Ishaku, Adamu, Bashir Saidu Musa, Ayuba Sanda, and Abubakar Muhammad Bakoji. 2018. "Mathematical Assessment of Social Media Impact on Academic Performance of Students in Higher Institution." 14(1):72–79. doi: 10.9790/5728-1401017279.

Juhari, Alisah. E, et all. 2024. "Optimal Control of a Modified Mathematical Model of Social Media Addiction." 6(2):112–23. doi: 10.15408/inprime.v6i2.41438.

Kamal, Mostofa, Mostak Ahmed, and Md Abu Hanif Sarkar. 2025. "Optimal Control Strategies for Mitigating Drug Addiction: A Mathematical Modeling Approach." Results in Control and Optimization 19(May):100580. doi: 10.1016/j.rico.2025.100580.

Karimah, P., & Subhan, M. (2022). Analisis stabilitas dan kontrol optimal model matematika kecanduan game online. Journal of Mathematics UNP, 7(3), 99–109.Khajji, Bouchaib, Abdelfatah Kouidere, Omar Balatif, and Mostafa Rachik. 2020. "Mathematical Modeling, Analysis and Optimal Control of an Alcohol Drinking Model with Liver Complication." Communications in Mathematical Biology and Neuroscience 2020:1–29. doi: 10.28919/cmbn/4553.

KOCABIYIK, M. 2025. "A Maple Program to the Analysis of Equilibrium Points in Social Media Addiction Model." 18(1):115–28. doi: 10.18185/erzifbed.1514507.

Lestari, Natasya Dyahayu, Abi Rizka, Yulia Arimanda, and Gina Agisna Nisa. 2025. "JSN : Jurnal Sains Natural Simulasi SIR Kecanduan Media Sosial Mahasiswa FMIPA UNRAM Dengan Metode Euler Dan Heun (SIR Model Simulation of Social Media Addiction among FMIPA UNRAM Students Using Euler and Heun Method)." (2).

Ma, Shuang Hong, Hai Feng Huo, and Xin You Meng. 2015. "Modelling Alcoholism as a Contagious Disease: A Mathematical Model with Awareness Programs and Time Delay." Discrete Dynamics in Nature and Society 2015. doi: 10.1155/2015/260195.

Pagga, Musdalifa, Syamsuddin Toaha, Uji Sensitivitas, Analisis Kestabilan, and A. Pendahuluan. 2015. "MODEL MATEMATIKA KECANDUAN MEDIA SOSIAL." 7:46–60.

Pellegrino, Alfonso, Alessandro Stasi, and Veera Bhatiasevi. 2022. "Research Trends in Social Media Addiction and Problematic Social Media Use: A Bibliometric Analysis." Frontiers in Psychiatry 13(1). doi: 10.3389/fpsyt.2022.1017506.

Przybylski, Andrew K., Kou Murayama, Cody R. Dehaan, and Valerie Gladwell. 2013. "Motivational, Emotional, and Behavioral Correlates of Fear of Missing Out." Computers in Human Behavior 29(4):1841–48. doi: 10.1016/j.chb.2013.02.014.

Romlah, Siti, Muhammad Thahiruddin, and Luluk Sarifah. 2025. "Solusi Numerik Model Matematika Pada Kasus Kecanduan Media Sosial Tiktok Di Pondok Pesantren Annuqayah Latee II Menggunakan Metode Runge Kutta." 1–8.

Sharma, Swarnali, and G. P. Samanta. 2013. "Drinking As an Epidemic: A Mathematical Model With Dynamic Behaviour." Journal of Applied Mathematics & Informatics 31(1_2):1–25. doi: 10.14317/jami.2013.001.

Shutaywi, M., Rehman, Z. U., Shah, Z., Vrinceanu, N., Jan, R., Deebani, W., & Dumitrascu, O. (2023).

Modeling and analysis of the addiction of social media through fractional calculus. Frontiers in Applied Mathematics and Statistics, 9, Article 1210404.Side, Syafruddin, Wahidah Sanusi, and Khaerati Rustan. 2020. “Model Matematika SIR Sebagai Solusi Kecanduan Penggunaan Media Sosial.” 3(2):126–38.

Wang, Xun Yang, Hai Feng Huo, Qing Kai Kong, and Wei Xuan Shi. 2014. “Optimal Control Strategies in an Alcoholism Model.” *Abstract and Applied Analysis* 2014. doi: 10.1155/2014/954069.

Widayati, Ratna, and Intrada Reviladi. 2023. “Analisa Kestabilan Bebas Kecanduan Pada Penyebaran Penggunaan Media Sosial Berdasarkan Model SEARQS.” *PYTHAGORAS Jurnal Pendidikan Matematika* 18(1):37–47. doi: 10.21831/pythagoras.v18i1.59556.

This page is intentionally leftblank

10.12928/bamme.v5i2.14795