

Transforming Cartesian coordinates into parallel coordinates in 2D space for geometric visualization

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Abstract

This study develops, step by step, the geometric transformation that carries the conventional two-dimensional Cartesian plane into a parallel coordinate system. Cartesian coordinates, while foundational in geometry and mathematics, are inherently limited to three dimensions, restricting their effectiveness in representing multidimensional data. As a solution, the parallel coordinate system, originally introduced by Maurice d'Ocagne and later developed into a coordinate system for multidimensional geometry by Alfred Inselberg, is explored for its ability to visualize data with higher dimensions using equidistant, parallel axes. The transformation is carried out in two explicit steps: a 90° rotation of the X-axis about the origin, so that it comes to coincide with the Y-axis, followed by a translation that separates the two axes by a fixed distance d . The duality principle is then applied to illustrate the reversed roles of points and lines. Two results follow. Every point $P(x^1, x^2)$ of the Cartesian plane is mapped to a line segment joining the two parallel axes; and every line $y = mx + b$ with $m \neq 1$ is mapped to the single point $(\frac{d}{(1-m)}, \frac{b}{(1-m)})$, while in the singular case $m = 1$ the dual is an ideal point of the projective extension of the plane. Visual constructions and worked examples validate the model, showing that the transformation preserves collinearity and the ratios of segments along a line while enabling multidimensional insights. This research highlights the potential of parallel coordinates in mathematical visualization, particularly in representing relationships among multiple variables in two-dimensional spaces.

Keywords: coordinate transformation, duality, geometric visualization, parallel coordinates

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INTRODUCTION

Mathematics, often regarded as the universal language of science, encompasses a plethora of branches, including geometry, algebra, calculus, and statistics. Its fundamental role lies in providing tools for understanding and modeling the physical and abstract worlds. Specifically, geometry serves as a meaningful means of object representation, enabling the description and analysis of spatial relationships and structures across various fields of study, including physics, biology, and engineering (McInnes et al., 2018; Meehan et al., 2019). Within this context, the importance of accurately and effectively representing multidimensional objects cannot be overstated, as it directly impacts our ability to analyze complex systems.

The Cartesian coordinate system, established by René Descartes in the 17th century, has provided a robust framework for understanding space in one, two, and three dimensions (Kobak & Berens, 2019). The Cartesian approach allows for the precise localization of points in space, represented as ordered pairs (2D) or triplets (3D) (Urpa & Anders, 2019). While this system has yielded significant insights and mathematical advancements, it also bears inherent limitations,

especially when applied to high-dimensional spaces. Notably, as the number of dimensions increases, the sparsity of the space becomes problematic, complicating the visualization and analysis of multidimensional data (Mazher, 2020; W. Zhang & Di, 2018). The inability of Cartesian coordinates to effectively encapsulate relationships among data points in more than three dimensions necessitates alternative visualization solutions to facilitate the comprehension of complex datasets (Smets et al., 2021).

The challenge of visualizing high-dimensional data has propelled the exploration of multidimensional representations beyond Cartesian frameworks. A seminal innovation in this regard is the parallel coordinate system, first proposed by Maurice d'Ocagne in 1885 (d'Ocagne, 1885) and later developed into a systematic coordinate system for multidimensional geometry by Alfred Inselberg (Inselberg, 1985, 2009; Inselberg & Dimsdale, 1990; Wegman, 1990). This method revolutionizes the way multidimensional data is visualized by allowing each data dimension to be represented as a parallel axis (Dertinger et al., 2024; Mitku et al., 2020). In this configuration, data points can be depicted as polylines, providing a comprehensive visualization tool that enables the analysis of relationships between variables without the spatial constraints imposed by Cartesian systems (Çakır et al., 2020). This advancement has profound implications for areas such as data science and bioinformatics, where the complexity of the information requires intuitive representation for thorough analysis (Gadekallu et al., 2020).

One of the key advantages of parallel coordinates is the ability to discern intricate relationships among multiple variables effectively. By mapping high-dimensional data onto a two-dimensional plane, researchers can identify patterns and trends that are often obscured in traditional Cartesian plots. This visualization technique facilitates the interpretation of multidimensional relations, allowing for clearer insights into data correlations and variability (Chushig-Muzo et al., 2022; Toprasertpong et al., 2022). Such insights are increasingly vital in domains where multiple variables must be evaluated simultaneously, such as economics, social sciences, and environmental studies, where the interplay of numerous factors can significantly impact outcomes (Eckelt et al., 2023; Konopka et al., 2021).

In real-world applications, the utility of parallel coordinates has expanded into various domains. For example, in the fields of bioinformatics and genomics, where vast datasets elucidate human health and disease characteristics, parallel coordinates allow researchers to visualize complex genetic interactions (Spathis et al., 2019; Stringer et al., 2019). Similarly, in finance, where multiple economic indicators influence market trends, parallel coordinates enable economists to discern relationships among these indicators in their entirety (Kanai et al., 2018; Liu et al., 2023). Consequently, the adoption of parallel coordinates as a visualization tool has gained traction, particularly as researchers confront the challenges of interpreting high-dimensional data, advocating for its integration into standard analytical practices (Alcaide & Aerts, 2021; Wu et al., 2018).

Despite this widespread use, most of the literature treats parallel coordinates as a ready-made plotting technique and presents the parallel arrangement of the axes as a given convention rather than as the outcome of an explicit geometric construction. Introductory accounts rarely show how the parallel axes are obtained from the familiar Cartesian axes, why the spacing d between them may be chosen freely, or how the point–line duality that underlies the whole representation is actually derived. The conceptual problem addressed in this article is therefore the following: how can the two-dimensional parallel coordinate system be constructed from the Cartesian system through a sequence of elementary and verifiable transformations, and what exactly happens to points and lines under that construction? Accordingly, this study (i) derives the construction as a 90° rotation of the X-axis about the origin followed by a translation through a distance d , (ii) proves the point–line duality for lines $y = mx + b$ in the regular case $m \neq 1$, and (iii) resolves the singular case $m = 1$ by extending the Euclidean plane projectively. The account is intended to make the geometric foundation of parallel

coordinates explicit, so that it may serve both in teaching and as a starting point for further work in computational visualization.

RESEARCH METHOD

This study adopts a qualitative, analytical–descriptive design based on conceptual mathematical analysis; no empirical data were collected. The object of the analysis is the two-dimensional parallel coordinate system, and the argument proceeds by deductive reasoning supported throughout by geometric construction. The analysis is organised in three stages.

In the first stage, the parallel coordinate system is constructed from the Cartesian system by two elementary transformations of the plane: a rotation R of the X -axis through 90° about the origin $O(0, 0)$, and a translation T that separates the axes so that they come to lie a fixed distance d apart. Because R and T are isometries, each step is verified by checking that lengths, angles, and the ordering of the values carried by each axis are preserved. This construction is set out in the subsection below.

In the second stage, the mapping induced on geometric objects is derived. A point $P(x_1, x_2)$ of the Cartesian plane is mapped to the line segment joining the value x_1 on the first parallel axis to the value x_2 on the second; a line $\ell: y = mx + b$ is mapped to the common intersection of the images of its points. The dual of a line is obtained analytically: the images of its X -intercept and of its Y -intercept are two lines whose equations are determined and then solved simultaneously. The resulting statement is formulated as a proposition and proved, and the regular and the singular case are separated according to the value of the slope m .

In the third stage, the derivation is validated. Each general result is instantiated with a numerical example that is constructed independently in both systems, so that the analytic result and the geometric construction may be compared directly. The singular case, in which the analytic derivation breaks down, is treated separately by extending the Euclidean plane to the projective plane. The geometric objects used as cases are points and lines.

Transformation from Cartesian coordinate axes to parallel coordinate system

Before transitioning into higher dimensions, the construction of the parallel coordinate system begins with the Cartesian coordinate system to provide a foundational understanding. The process starts by exploring the two-dimensional coordinate system. Both the Cartesian and parallel coordinate systems share fundamental similarities, particularly in their basic components namely the abscissa and ordinate (X -axis and Y -axis). The following section outlines the construction of the two-dimensional parallel coordinate system based on the two-dimensional Cartesian coordinate system.

Identification of Cartesian coordinate axes

The Cartesian coordinate system consists of two perpendicular axes: the X -axis (abscissa) and the Y -axis (ordinate), with the origin at the point $O(0,0)$.

Rotation of the X -axis by 90°

The initial step in constructing the parallel coordinate system is to rotate the X -axis by 90° , centered at the origin O . As shown in Figure 1(b), the initially horizontal X -axis is rotated to become vertical, overlapping with the Y -axis. Since a rotation is an isometric transformation, one that preserves both lengths and angles, the scale carried by the axis is unaffected: its numerical values and their units remain exactly as they were in the Cartesian system. Above the origin O , the coordinate values remain positive, while values below it are negative consistent with Cartesian conventions. Thus, the rotated X -axis becomes a vertical axis that bisects the plane into two regions, preserving sign conventions analogous to the original Cartesian system.

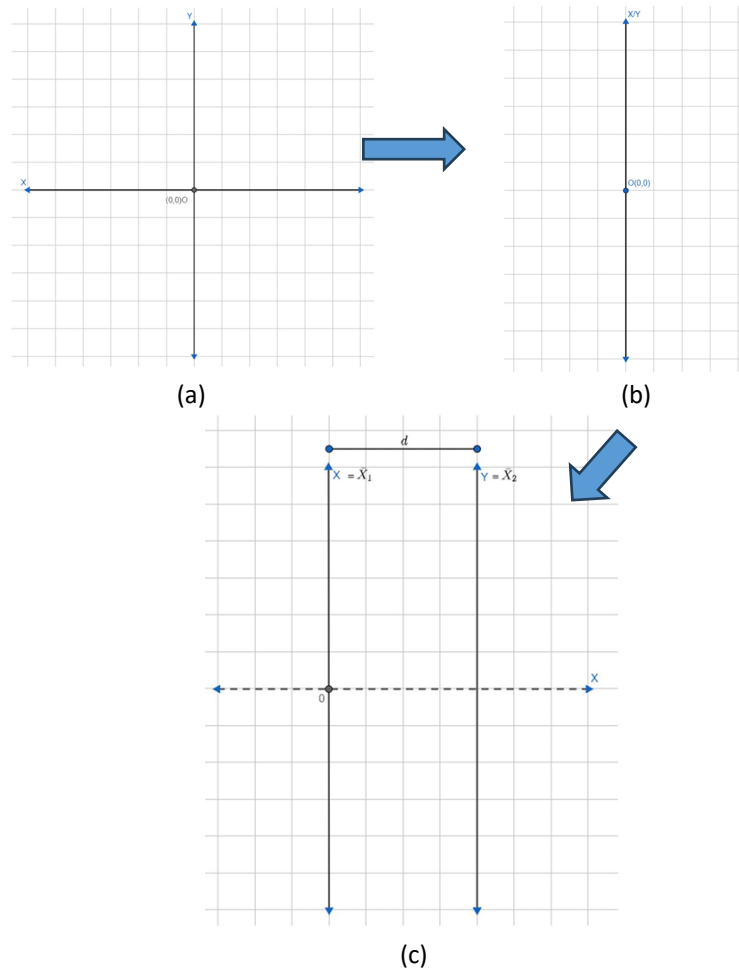


Figure 1. Two-dimensional Cartesian coordinates (a), rotation of the X-axis by 90° (b), and final construction from two-dimensional Cartesian into parallel coordinates (c).

Translation of the X-axis by a distance d

After the X-axis has been rotated through 90° so that it coincides with the Y-axis, the two superimposed axes are separated by a translation through a distance d . The rotated X-axis is held fixed at the origin and becomes the first axis of the parallel system, while the second axis is placed parallel to it at a distance d to its right; equivalently, the first axis comes to lie a distance d to the left of the second. This separates the previously overlapping axes and so forms the parallel coordinate system, enabling visual differentiation. The direction is not arbitrary. Placing the image of the X-axis on the left makes the two axes appear in the reading order $\bar{X}_1, \bar{X}_2$, which is the order in which the coordinates of a point are listed and the order that generalises directly to the n -dimensional layout $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n$ read from left to right. Holding the first axis at the origin also fixes a coordinate frame for the parallel plane, so that a point of that plane may be given ordinary Cartesian coordinates; this is what allows the dual of a line to be written in the compact form derived in the following section. Choosing the opposite direction would place the second axis at $x = -d$ and would simply reflect the whole construction in the vertical axis, replacing d by $-d$ in every formula without altering any of the geometry.

The rotated X-axis, now vertical, is renamed as $\bar{X}_1$, whereas the Y-axis from the Cartesian system is now represented as $\bar{X}_2$ in the parallel coordinate system. This transformation maintains consistent unit measurements from the Cartesian system, but reconfigures the axes

into a parallel layout with fixed spacing. This enables coordinate representation in the form of lines, which aligns with the principles of the parallel coordinate system.

RESULTS AND DISCUSSION

Transforming a point in Cartesian coordinates into parallel coordinates

In the Cartesian coordinate system, particularly in two dimensions, each point represents a specific location in space. Any point P in a two-dimensional plane can be represented by a coordinate pair (x_1, x_2) where x_1 denotes the value along the X-axis (abscissa), and x_2 denotes the value along the Y-axis (ordinate), as illustrated in Figure 2.

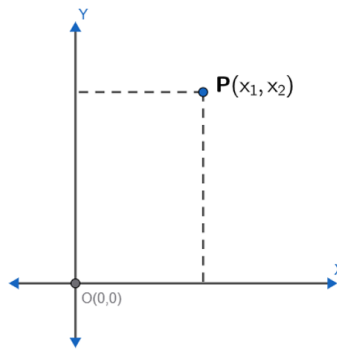


Figure 2. Representation of point $P(x_1, x_2)$

Figure 2 demonstrates how the point $P(x_1, x_2)$ is represented within the Cartesian coordinate system. The point is illustrated by projecting its coordinates onto the respective axes: x_1 on the X-axis and x_2 on the Y-axis. This point is then converted into the parallel coordinate system, in which the same point is represented by the notation $\bar{P}$. The point $\bar{P}$ lies on a line connecting the vertical axes $\bar{X}_1$ and $\bar{X}_2$, where x_1 is projected onto $\bar{X}_1$ and x_2 onto $\bar{X}_2$. This projection forms a polyline that visually expresses the position of point P in the parallel coordinate system, as depicted in Figure 3.

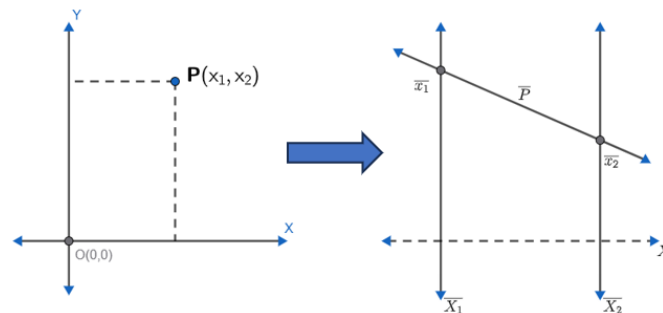


Figure 3. Conversion of point $P(x_1, x_2)$ into the parallel coordinate system.

This representation equally applies when point $P(x_1, x_2)$ is defined numerically. For instance, for the two-dimensional point $P(5,2)$, the parallel coordinate system represents the value $x_1 = 5$ on the first vertical axis $\bar{X}_1$ and the value $x_2 = 2$ on the second vertical axis $\bar{X}_2$ (See Figure 4).

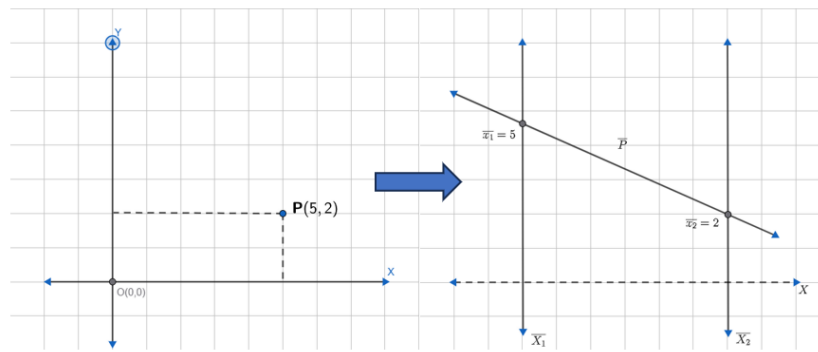


Figure 4. Conversion of point $P(5,2)$ from Cartesian to parallel coordinates

From this explanation, it can be concluded that lines in the parallel coordinate system are the representational outcomes of points in the two-dimensional Cartesian coordinate system. This transformation illustrates how each Cartesian point is transformed into a line segment in the parallel coordinate system, establishing a new relational structure between points and lines. This conceptual shift is fundamental in parallel coordinates, where a set of individual points translates into interconnected lines, enabling a different perspective for multivariate data visualization and analysis.

Duality of lines and points in Cartesian and parallel coordinates

In geometric analysis, the transformation between Cartesian coordinates and parallel coordinates provides an interesting view of the duality between lines and points. Each line in the Cartesian coordinate system, represented in the linear form $y = mx + b$ where m is the slope and b is the intercept, has a corresponding dual representation as a point in the parallel coordinate system. The derivation of this dual divides into two distinct cases, according to the value of the slope m . The first case applies when $m \neq 1$, and the second when $m = 1$.

Case 1: Line $l \equiv y = mx + b$ with $m \neq 1$

Assume the distance between the vertical axes in the parallel coordinate system is d . The dual of the line $y = mx + b$ (with $m \neq 1$) in parallel coordinates is a point given by $[\bar{1}(\frac{d}{1-m}, \frac{b}{1-m})]$. To prove this statement, consider any line $l \equiv y = mx + b$ with $m \neq 1$: setting $y = 0$ in the equation gives $0 = mx + b \Rightarrow x = \frac{-b}{m}$, meaning the X-intercept is $A(\frac{-b}{m}, 0)$; setting $x = 0$ in the equation gives $y = m(0) + b = b$, so the Y-intercept is $B(0, b)$.

The point $A(\frac{-b}{m}, 0)$ lies on the X-axis and $B(0, b)$ lies on the Y-axis in Cartesian coordinates. To construct the parallel coordinates, consider the lines connecting transformed positions.

Let us now determine the equations of the image lines $\bar{A}$ and $\bar{B}$ in the parallel coordinate system (See Figure 5). To avoid any confusion with the slope m of the original line l , the slopes of these two image lines are written m_A and m_B respectively. Line A : passes through $(d, 0)$ and $(0, \frac{-b}{m})$. The slope is in Equation (1).

$$m_A = \frac{\frac{-b}{m} - 0}{0 - d} = \frac{-\frac{b}{m}}{-d} = \frac{b}{md} \quad (1)$$

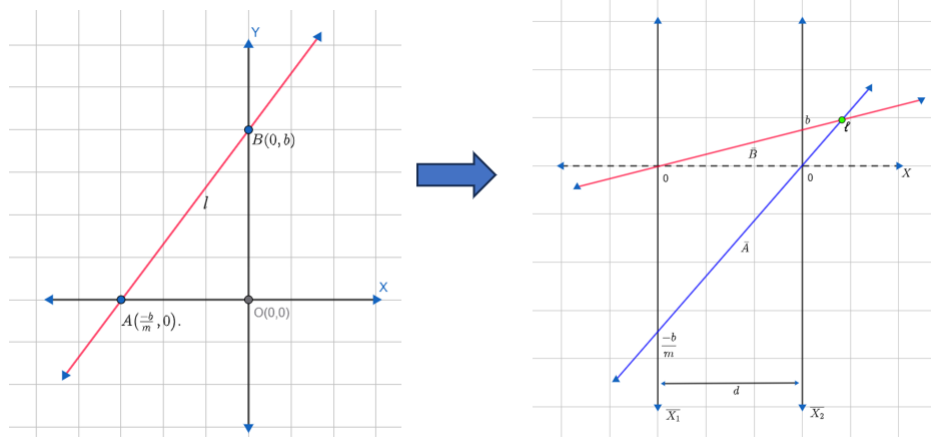


Figure 5. Proof of $\bar{I}\left(\frac{d}{1-m}, \frac{b}{1-m}\right)$

Using point $(d, 0)$ to find the intercept $0 = \frac{b}{md} \cdot d + c \Rightarrow c = -\frac{b}{m}$. Hence, the equation of line A is:

$$y = \frac{b}{md}x - \frac{b}{m} \quad (2)$$

Line B: passes through $(0,0)$ and (d, b) with slope: $m_B = \frac{b-0}{d-0} = \frac{b}{d}$. So its equation is:

$$y = \frac{b}{d}x \quad (3)$$

Equating the two line equations, $\frac{b}{md}x - \frac{b}{m} = \frac{b}{d}x$, which simplify $\frac{b}{md}x - \frac{b}{d}x = \frac{b}{m}$, or $x \cdot \left(\frac{b}{d} \left(\frac{1}{m} - 1\right)\right) = \frac{b}{m}$. Factoring further gives $x \cdot \left(\frac{b}{d} \cdot \frac{1-m}{m}\right) = \frac{b}{m}$, so $x \cdot \left(\frac{1-m}{d}\right) = 1$, and therefore $x = \frac{d}{1-m}$. Substitute $x = \frac{d}{1-m}$ into equation (3) to find $y = \frac{b}{d} \cdot \frac{d}{1-m} = \frac{b}{1-m}$.

Hence, the intersection point is: $\bar{I}\left(\frac{d}{1-m}, \frac{b}{1-m}\right)$. Thus, it is proven that if the vertical axis distance in the parallel coordinate system is d , then the dual of the line $y = mx + b$ with $m \neq 1$ is a point at: $\left(\frac{d}{1-m}, \frac{b}{1-m}\right)$.

Two boundary situations are worth recording. If $m = 0$ the line ℓ is horizontal and has no X -intercept, so the auxiliary point A used above does not exist; applying the same argument to any two distinct points of ℓ nevertheless yields the dual, and the formula returns the point (d, b) , which lies on the axis $\bar{X}_2$ at height b . Vertical lines $x = c$ are not of the form $y = mx + b$ and therefore fall outside the family treated here; the images of their points are the segments joining the fixed value c on $\bar{X}_1$ to every value on $\bar{X}_2$, and these form a pencil through the single point $(0, c)$ on $\bar{X}_1$.

Suppose a straight line l is given by the equation: $l: y = 2x - 1$, with the spacing of the vertical axes fixed at $d = 6$. This line passes through three points: $A(0, -1)$, $B(1, 1)$, $C(2, 3)$ (See Figure 6).

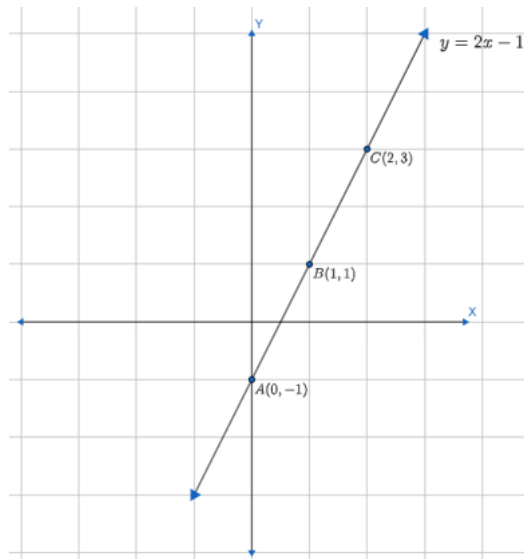


Figure 6. Line $y = 2x - 1$

These three points lie on the line l . Next, when these points are constructed in the parallel coordinate system, each point is represented as a line segment (See Figure 7).

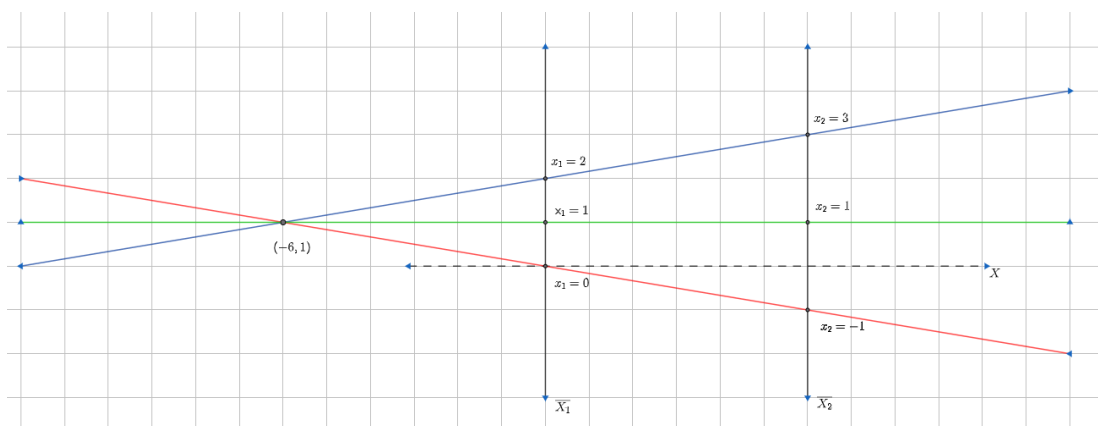


Figure 7. Parallel coordinate form line $y = 2x - 1$.

The extensions of these segments intersect at the point: $l'(-6, 1)$. This agrees with the general result, since $d/(1 - m) = 6/(1 - 2) = -6$ and $b/(1 - m) = -1/(1 - 2) = 1$. The intersection point is thus the outcome of the geometric transformation in the parallel coordinate system.

Case 2: Line $l \equiv y = mx + b$ with $m = 1$

In the standard coordinate system, a line can be represented by the equation $y = mx + b$. In the context of parallel coordinates, the duality transformation for this line yields a point. However, this transformation encounters a singularity when the slope m approaches 1. The transformation involves expressions that are dependent on the term $(1 - m)$ in the denominator, such as: $\frac{d}{1-m}$ and $\frac{b}{1-m}$. As $m \rightarrow 1$, the denominators of these expressions approach zero, causing the resulting coordinate values to approach infinity. This invalidates the standard duality rule for lines with a slope of 1.

This singularity is removed by working in the projective plane rather than in the Euclidean plane. Projective geometry is the study of those properties of figures that remain invariant under

projection. Its real projective plane is obtained from the Euclidean plane by adjoining one new point to each family of mutually parallel lines, with the effect that any two distinct lines meet in exactly one point and the exceptional case of parallelism disappears (Richter-Gebert, 2011). Each adjoined point is called an ideal point, or a point at infinity. It is not a point of the Euclidean plane but a direction, shared by every line of a given slope, and the set of all such points forms the line at infinity.

With this extension the duality principle becomes exhaustive. Write $P^\infty(k)$ for the ideal point common to all lines of slope k . As $m \rightarrow 1$ the dual point $(d/(1-m), b/(1-m))$ recedes without bound along the direction of the vector (d, b) , that is, along the direction of slope b/d ; and this is precisely the common direction of the image lines of the points of ℓ , since the point $(t, t+b)$ is carried to the segment joining $(0, t)$ to $(d, t+b)$, whose slope is b/d for every value of t . The dual of the line $y = x + b$ is therefore the ideal point $P^\infty(b/d)$. Every line of the Cartesian plane accordingly has exactly one dual in the projective extension of the parallel coordinate plane: an ordinary point when $m \neq 1$, and an ideal point when $m = 1$. This formulation provides a consistent and comprehensive framework for the duality between points and lines in parallel coordinates.

Line $l: y = x - 1$. The line l has a slope of 1 and a vertical shift of -1 . The points on this line are: $(-2, -3)$, $(-1, -2)$, $(0, -1)$, $(1, 0)$, $(2, 1)$ (See Figure 8).

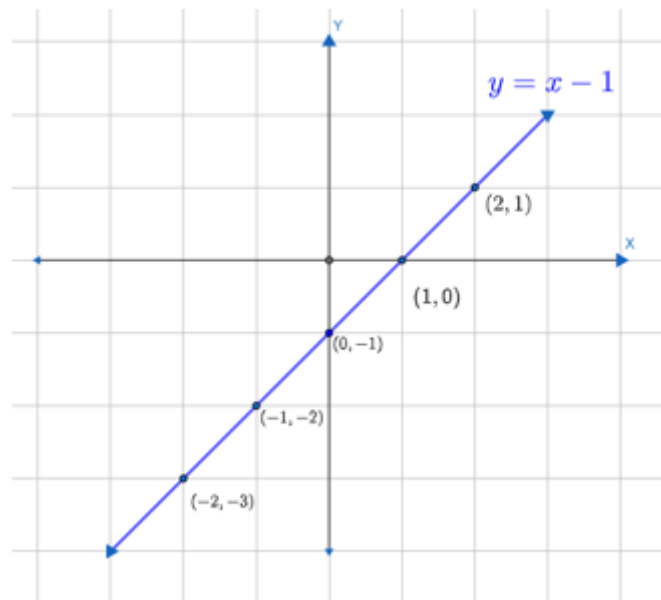


Figure 8. Line $y = x - 1$

When these points are constructed in parallel coordinates, they form a set of parallel lines. These image lines all have the same gradient, namely $b/d = -1/d$. They therefore do not intersect anywhere in the Euclidean plane: no two of them share a point, and however far they are extended they remain a fixed distance apart. In the projective extension of the plane, however, they do share exactly one point, the ideal point $P^\infty(-1/d)$ at which every line of gradient $-1/d$ meets (See Figure 9). This is what is meant by saying that the images meet at infinity: the meeting takes place at an ideal point of the projective plane, and not at any point of the Euclidean plane.

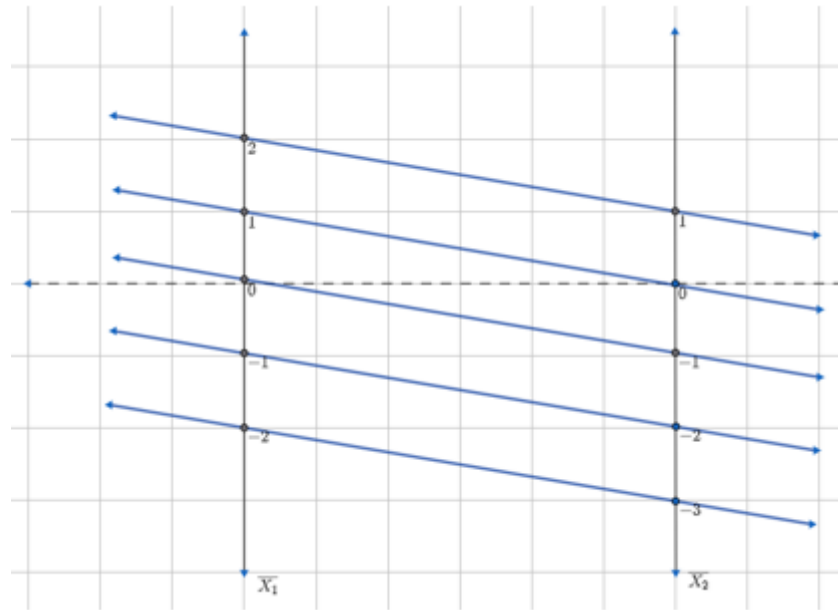


Figure 9. Parallel coordinate form line $y = x - 1$

CONCLUSION

The Cartesian coordinate system faces limitations in handling multidimensional data, prompting the development of the Parallel Coordinate System (PCS) as a more effective alternative. By arranging the variable axes in parallel, the PCS turns complex data into interconnected lines and so makes patterns easier to recognise. The construction was obtained here from the Cartesian system by two elementary transformations, a 90° rotation of the X-axis about the origin followed by a translation separating the axes by a distance d , and the correspondence it induces was shown to be dual: points of the Cartesian plane become line segments, while a line $y = mx + b$ becomes the single point $(d/(1 - m), b/(1 - m))$ when $m \neq 1$, or an ideal point of the projective plane when $m = 1$.

The parameter d governs the spacing of the axes, and its role should be stated precisely. The transformation is not a congruence of the Euclidean plane: it does not preserve the distance between arbitrary points, nor the angle at which two lines meet, and the visual shape of a configuration changes as d changes. What it does preserve are the properties on which the representation actually depends, namely collinearity, incidence, and the ratio in which a point divides a segment. Altering d rescales the horizontal direction of the parallel plane and therefore moves the dual point, but it leaves the dual correspondence itself intact, so that the structural information carried by the diagram is unaffected.

These results have implications in two directions. For mathematics education, the construction offers a route into multidimensional visualization that requires nothing beyond rotation, translation, and the equation of a straight line, and it gives the abstract notion of a point at infinity a concrete form, as the limit of a dual point that recedes as $m \rightarrow 1$; the duality itself provides students with an accessible first encounter with the reciprocity between points and lines that is central to projective geometry. For computational visualization, the closed form of the dual point supplies an exact criterion for collinear structure in parallel coordinate plots, since a set of Cartesian points is collinear precisely when the corresponding segments in the parallel plane share a common intersection. The present treatment is confined to two dimensions and to straight lines; extending the derivation to n parallel axes, and to conics and other non-linear objects, is a natural continuation of this work.

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DECLARATION

Author contribution

All authors contribute in the research and/or writing the paper, and approved the final manuscript.

<i>Gibran Khalil Gibran</i>	Conceptualizing the research idea, leading the investigation, setting up the methodology, analyzing the data, and writing the first draft.
<i>Farida Nurhasanah</i>	Assisting the investigation, reviewing the validity of the methodology, and enriching the data analysis.

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Conflict of interest

All authors declare that they have no competing interests.

Ethics declaration

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The use of artificial intelligence

We do not use any generative AI tools to write any part of this paper.

Additional information

Not available.

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