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Simulating Bitcoin price movements with the Bates model and Monte Carlo methods

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ABSTRACT

This study investigates the price dynamics of Bitcoin, a highly volatile and speculative digital asset. Using daily closing price data from January 2023 to January 2024, we apply the Bates model, which combines stochastic volatility with jump-diffusion processes, to better capture both continuous fluctuations and sudden, large price changes in the market. The model parameters are calibrated using historical data and evaluated through Monte Carlo simulation with 10,000 generated price paths over a 31-day forecast horizon. The results demonstrate a strong short-term predictive performance, with a Mean Absolute Percentage Error (MAPE) of 4.32%. This indicates that the Bates model can capture both volatility clustering and abrupt shifts, which are characteristic of Bitcoin. The findings suggest that this approach provides a valuable tool for risk management and investment decision-making in highly uncertain and dynamic markets.

Introduction

Volatility is a central indicator of uncertainty and risk in financial markets, playing a vital role in asset pricing, portfolio management, and trading strategies (Rustamov, 2024; Sharif et al., 2023). In the context of cryptocurrencies, especially Bitcoin, volatility is even more pronounced. Influenced by market sentiment, regulatory developments, and technological evolution, Bitcoin exhibits non-linear price dynamics characterized by abrupt swings and clustering volatility (Brini & Lenz, 2024; Narayan & Kumar, 2024).

Traditional models such as those assuming constant volatility like the Geometric Brownian Motion (GBM) are often insufficient for capturing the empirical features of Bitcoin prices. These models generally fail to reflect the high kurtosis, skewness, and frequent large jumps observed in crypto markets(AlMadany et al., 2024; Kim et al., 2021; Petropoulos et al., 2022). As such, there is a growing need for more sophisticated frameworks that can accommodate both continuous and discontinuous price behavior in highly speculative assets.

The Bates model, an extension of the Heston stochastic volatility model integrated with

Merton's jump-diffusion process, addresses this limitation by incorporating both stochastic variance and Poisson-distributed jumps (Levendis, 2023; Sene et al., 2021). This hybrid approach makes the Bates model particularly well-suited for modeling assets like Bitcoin, which frequently exhibit volatility clustering along with sudden and sharp price movements (Chen et al., 2024). Unlike classical models, it provides a richer and more flexible structure for simulating realistic market conditions.

Despite its advantages, the application of the Bates model in cryptocurrency research remains limited, especially when compared to traditional asset classes. Most studies either simplify the model or do not fully explore the role of jump dynamics in capturing the behavior of digital assets (Singh et al., 2024; Wati et al., 2024). This highlights the need for empirical studies that rigorously apply and calibrate the Bates framework to actual Bitcoin market data.

This study offers three key contributions. First, it calibrates the Bates model using one full year of daily Bitcoin closing price data, enabling the parameters to reflect asset-specific behavior. Second, it utilizes Monte Carlo simulation to generate 10,000 future price paths over a 31-day forecast period, providing a probabilistic framework to assess price uncertainty. Third, it analyzes the frequency and magnitude of jumps, offering insight into Bitcoin's exposure to tail risks. These elements distinguish this work from prior studies and strengthen the relevance of the Bates model in crypto-finance research.

By capturing both stochastic volatility and discrete price jumps, this study aims to provide a more accurate and realistic tool for modeling Bitcoin price movements. The findings have practical implications for investors, risk managers, and financial analysts operating in highly volatile digital asset markets.

Method

The historical price of Bitcoin was collected from Yahoo Finance for this study. Daily closing prices for a year, from January 1, 2023, to January 31, 2024, are included in the dataset.

Outliers and missing data are checked in the raw data. Outliers will be eliminated, and linear interpolation will be used to manage the missing points. Equation 1 will be used to get the log return (r_t) for Bitcoin, where P_t represents the price at time t:

$$r_t = \log \frac{P_t}{P_{t-1}} \quad (1)$$

Because it stabilizes the variance of returns over time, this transformation produces a stationary series which assists in further statistical analysis and volatility modeling.

The Bates model connects both stochastic volatility and jump diffusion to reflect the unique price dynamics of Bitcoin. It is a modification of the Heston stochastic volatility model and the Merton Jump Diffusion model. The Bates model in logarithmic terms may be obtained from the Merton and Heston models using Itô's Lemma (equation 2) in the process shown below (Chiarella et al., 2015; Levental et al., 2013):

$$d\big(f(S_t)\big) = f'(S_t)dS_t + \frac{1}{2}f''(S_t)(dS_t)^2 \quad (2)$$

In order to account for unexpected, fluctuating price fluctuations, the Merton Jump Diffusion model (equation 3) adds jumps to the log-return process. It uses a jump term characterized by a Poisson process and a Brownian motion component for continuous price evolution to simulate the asset price S_t. The concept of the model is:

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$$d(\ln(S_t)) = \left(\mu - \frac{\sigma^2}{2} - \lambda E[J-1]\right)dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t} \ln J_i\right) \tag{3}$$

where μ is the drift, σ represents the volatility, dW_t is a Wiener process, λ is the jump intensity, J represents the magnitude of jumps, and N_t is Poisson process counting the number of jumps up to time t.

By defining volatility as a stochastic process that follows to a mean-reverting square-root process, the Heston model (equation 4) improves the framework. This captures the clustering effects frequently observed in financial markets and enables the variance v_t to change dynamically. The logarithmic expression for the Heston model is:

$$d(\ln(S_t)) = \left(r - \frac{v_t}{2}\right)dt + \sqrt{v_t}dW_t^S \tag{4}$$

where r is the risk-free rate, v_t is the stochastic variance, and dW_t^S is a Brownian motion component.

These two methods are put together in the Bates model (equation 5), integrating the jump component from the Merton model as well as the stochastic variance from the Heston model. In logarithmic terms, the Bates model is expressed as follows:

$$d(\ln(S_t)) = \left(r - \frac{v_t}{2} - \lambda E[J-1]\right)dt + \sqrt{v_t}dW_t^S + d\left(\sum_{i=1}^{N_t} \ln J_i\right) \tag{5}$$

This formulation offers an additional framework for the price dynamics of Bitcoin by capturing both unexpected, discrete price fluctuations and recurring price changes.

Key parameters in the Bates model—drift (μ), volatility (σ), mean reversion rate (κ), long-term variance (θ), and volatility of variance (σ_v)—are calibrated using Maximum Likelihood Estimation (MLE). The mean reversion rate κ and long-term variance θ are estimated from the autocorrelation and variance of returns, capturing the tendency of volatility to revert to a stable mean level. The volatility of variance σ_v is determined by analyzing daily variance changes, then annualized to account for year-over-year fluctuations.

To estimate potential future price paths for Bitcoin, a Monte Carlo simulation is implemented using the Python programming language, with core numerical computations carried out via the NumPy and SciPy libraries. The simulation process follows a structured pipeline based on the Bates model, which integrates stochastic volatility and jump-diffusion elements.

The simulation proceeds as follows:

(1) Initialize Parameters: Set the calibrated values of the model parameters, including drift μ, long-term variance θ, mean reversion rate κ, volatility of volatility σ_v, correlation ρ, jump intensity λ, mean jump size μ_J, and jump size volatility σ_J.

(2) Discretize the Time Horizon: Define the forecast horizon (31 days) and discretize it into daily intervals.

(3) Simulate Stochastic Variance v_t: Use the Euler-Maruyama scheme to simulate the Heston variance process as a mean-reverting square-root diffusion.

(4) Generate Jump Events: For each time step, draw from a Poisson distribution with intensity λ to determine if a jump occurs.

(5) Simulate Jump Sizes: When a jump is triggered, simulate the jump size from a lognormal distribution using μ_J and σ_J.

(6) Simulate Asset Paths S_t: Combine the stochastic variance and jump components to simulate 10,000 asset price paths using the log-transformed Bates model dynamics.

(7) Aggregate Forecasts: Average all simulated paths at each time step to generate the expected future price trajectory.
(8) Evaluate Accuracy: Compare the simulated price path with actual Bitcoin prices using the Mean Absolute Percentage Error (MAPE) as the performance metric.

This Monte Carlo approach enables the incorporation of both persistent volatility and rare but impactful price jumps, offering a probabilistic framework for forecasting Bitcoin prices under different market conditions. It is particularly useful for risk assessment and scenario analysis in high-volatility environments such as cryptocurrency markets.

Results and discussion

The results of the study provide insights into Bitcoin's dynamic price behavior by considering both continuous and discrete changes with implementation of the Bates model. As seen in Figure 1, the dataset of daily closing prices for Bitcoin from January 1, 2023, to January 31, 2024, shows significant fluctuation and a wide range of price swings, reflecting the market's volatile nature.

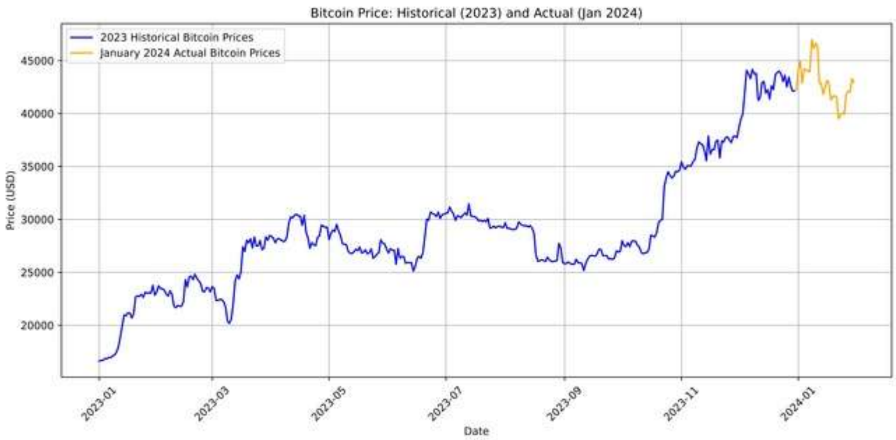

Figure 1. Bitcoin price from 1 January 2023 to 31 January 2024

The market for Bitcoin was very unpredictable over this time, as seen by the mean price of \$28,822.62 with a standard deviation of \$5,865.94. With a standard deviation of 0.0227 and a mean log return of 0.0026, the daily price fluctuations were significant and prevalent. A positive skew is indicated by skewness values of 0.69 for log returns and 0.83 for prices, which point to an asymmetric distribution with a tendency for extremely positive price fluctuations. Since the kurtosis of 3.00 and 0.73 for prices approximates a normal distribution, the return distribution of Bitcoin does not show excessive tails over this time frame. However, this volatility pattern still emphasizes the necessity of a model such as the Bates model, which can account for the stochastic volatility and price spikes that are unusual to Bitcoin and provide a precise understanding of its price dynamics.

Figure 2 displays the daily log returns of Bitcoin throughout 2023. The plot highlights the highly volatile nature of Bitcoin, characterized by frequent spikes and reversals, especially during the first and last quarters of the year. These fluctuations are typical of cryptocurrency markets, where sentiment-driven movements and external shocks often result in large, abrupt price changes. To assess the statistical properties of this return series, the Augmented Dickey-Fuller (ADF) test was performed. The test yielded an ADF statistic of –18.71 with a p-value of 2.04×10^{-30}, which is significantly lower than all conventional critical values (1%, 5%, and 10%). This

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provides strong evidence to reject the null hypothesis of a unit root, confirming that the return series is stationary. This stationarity is essential for the application of time-series models like the Bates model, which assumes that returns exhibit stable statistical properties over time. The stationarity of the series further justifies the use of stochastic modeling techniques for forecasting and risk analysis in the volatile cryptocurrency domain.

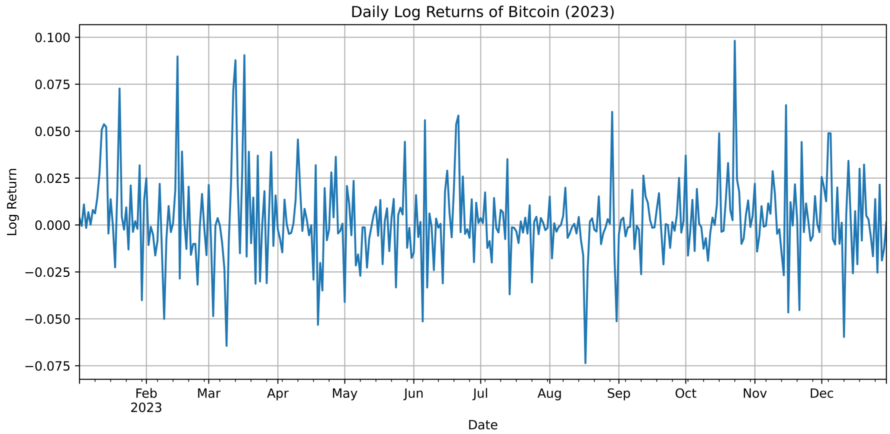

Figure 2. Bitcoin log return from 1 January 2023 to 31 December 2023

The model's responsiveness to short-term volatility and current market circumstances are reflected in the initial variance ($v_0 = 0.00033$), which is calculated from the previous 14 days of log returns. The model was able to react quickly to the current volatility patterns in the Bitcoin market because of the relatively low initial variance, which suggests that recent movements were gentle. A baseline level around which variance reverts is provided by the long-term variance ($\theta = 0.00051$), which is obtained from the complete dataset. This figure captures Bitcoin's tendencies for both stable and high-volatility periods, indicating a somewhat greater average volatility level for the whole time period.

The rates at which volatility returns to its long-term mean is represented by the mean reversion rate ($\kappa = 713.72$). A high number indicates that Bitcoin's volatility is subject to sudden shifts, with periods of extreme volatility rapidly returning to normal levels. This metric is consistent with the volatility clustering that has been seen in Bitcoin, when periods of mild activity tend to be followed by sudden price fluctuations. The variation in the variance itself is captured by the volatility of variance ($\sigma_v = 0.0295$), which is known as the "volatility of volatility." The model is able to represent both stable and chaotic market times as a result to this adequate level, which shows that Bitcoin's volatility variations are notable but not improperly unpredictable.

Bitcoin's log returns and variance changes have a moderately positive association ($\rho = 0.2679$), indicating that price rises are relatively associated with increased volatility. The market's view that Bitcoin's price frequently rises with greater uncertainty, which is characteristic of speculative assets, is supported by this correlation. Because price and volatility are not viewed as separate variables, their interconnectedness gives the simulated trajectories more realism.

In the simulation, the drift ($\mu = 0.94249$), which is calculated as the annualized mean of Bitcoin's daily log returns, shows an increasing trend that corresponds to the cryptocurrency's historical growth rate throughout the period under study. This drift, along with the risk-free rate ($r = 4.3\%$), enables the model to account for risk-neutral circumstances in the pricing framework and represent Bitcoin's long-term upward tilt.

The Bates model's jump component adds factors that explain Bitcoin's abrupt price fluctuations. The jump intensity ($\lambda_J = 6.92$) indicates that the market for Bitcoin regularly sees notable changes from normal price fluctuations, with a projected average of almost seven leaps a year. This is consistent with Bitcoin's history of significant and sudden fluctuations, which are frequently brought on by macroeconomic, regulatory, or market sentiment. The tendency for sudden price increases or losses in the Bitcoin market is captured by the mean jump size ($\mu_J = 0.0624$), which shows that each jump is normally connected with a 6.24% price shift. Furthermore, the jump size volatility ($\sigma_J = 0.0608$) shows that although jumps occur often, their magnitudes vary, enabling the model to account for a range of potential price implications, from less serious to severe.

Monte Carlo simulations of Bitcoin price pathways were conducted over a 31-day period using the calibrated parameters of the Bates model, generating 10,000 possible outcomes that incorporate both continuous stochastic volatility and discrete jump components. As illustrated in Figure 3, the orange line represents the forecasted mean price trajectory, while the shaded region indicates the 95% confidence interval derived from the simulated paths. The blue line shows the actual Bitcoin prices during January 2024, which largely fluctuate within the confidence bounds, demonstrating the model's ability to realistically capture the dual nature of Bitcoin's price behavior, which are continuous variance shifts and sudden market movements. The widening of the confidence interval over time reflects the increasing uncertainty in longer-term forecasts, further validating the probabilistic strength of the Bates model in dynamic market conditions.

The Mean Absolute Percentage Error (MAPE), which was used to assess the model's accuracy, came out to be 4.32% and its daily distribution displayed in Figure 4. Although some divergence was seen during times of intense market volatility, this finding indicates that the Bates model has an acceptable level of predictive accuracy for short-term estimates. These variations imply that, although generally useful, the jump parameters could want some more fine-tuning to properly represent Bitcoin's behavior during sudden shifts in the market.

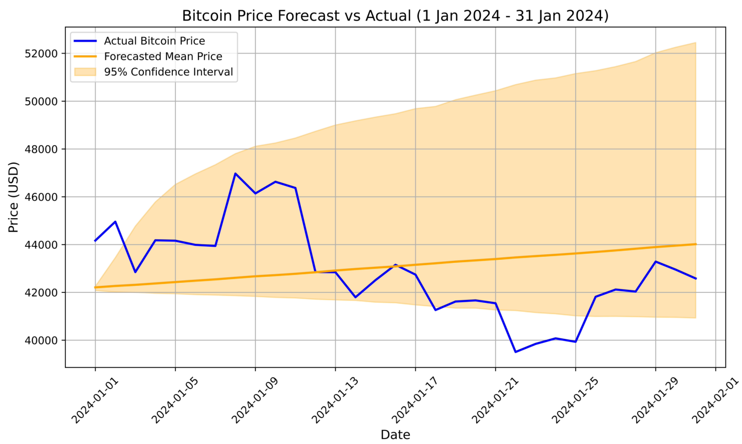

Figure 3. Bitcoin actual price and mean simulated price (1 January – 31 January 2024)

The Bates model's capacity to capture sudden and unusual price changes, which are characteristic of Bitcoin, is greatly improved by the jump component. The model reflects frequent price swings with an annual jump intensity of 6.92, which is consistent with the asset's vulnerability to sudden fluctuations brought on by outside factors.

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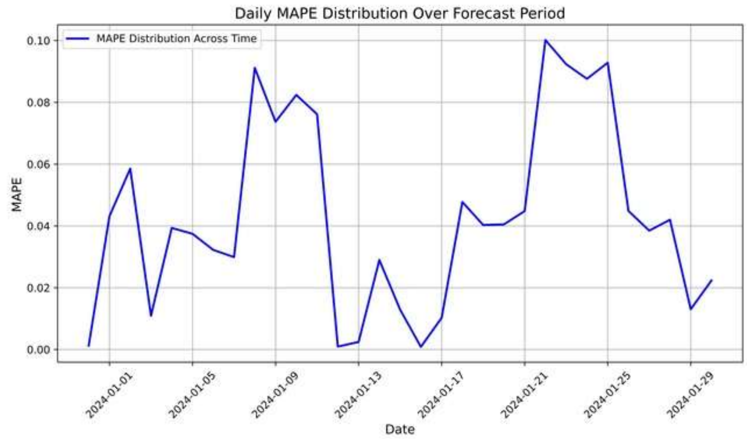

Figure 4. MAPE over forecast period

The average size of these jumps visualized in Figure 5 is essential, as indicated by the positive mean jump size of 6.24%, which is frequently influenced by media reports or changes in regulations. The model is made more flexible by the leap size variability, which is represented by a jump size volatility of 6.08%. This enables the model to capture both minor and large price movements. This part ensures that the model presents a true picture of tail risk and appropriately captures the sudden shifts that may occur in the Bitcoin market.

Bitcoin's dual nature of slow price fluctuations mixed with rapid shifts is well captured by the Bates model, which integrates stochastic volatility and jump-diffusion. The model is especially helpful for short- to medium-term forecasting in the extremely volatile Bitcoin market because of its dual capacity, which allows it to produce realistic price pathways that consider both continuous and discrete changes. By modeling a variety of alternative price pathways under various market situations, Monte Carlo simulations based on this model give a probabilistic estimate of prospective future prices, which is useful for risk management.

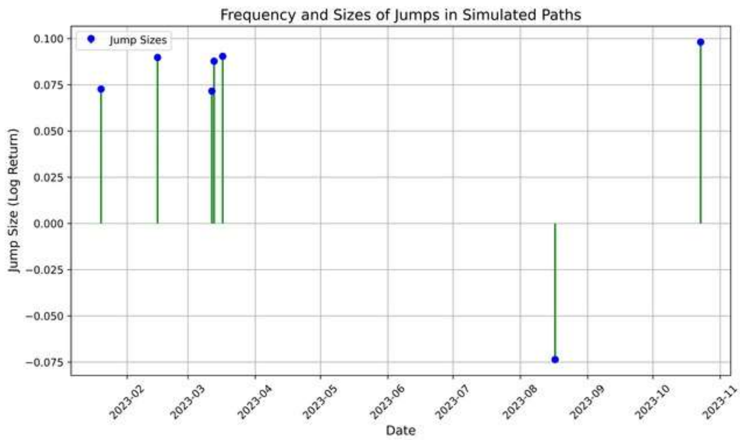

Figure 5. Jump frequency over simulation

According to the results, the Bates model does well in normal market circumstances, but it might be necessary to recalibrate it often to preserve accuracy in times of high market volatility. Because it captures both abrupt price surges and gradual volatility clusters, which are frequently missed in previous models, this model provides practitioners with an improved approach to forecasting and risk assessment. The incorporation of jump and stochastic variance components improves the model's robustness and offers a solid basis for risk management and wise investing choices in the volatile Bitcoin market.

Conclusion

By combining discrete jumps and continuous stochastic volatility, this study shows the capacity with which the Bates model captures the unique price dynamics of Bitcoin. The model reached a Mean Absolute Percentage Error (MAPE) of 4.32%, indicating good predictive accuracy for short-term projections, after being calibrated using daily Bitcoin prices from January 2023 to January 2024 and 10,000 Monte Carlo simulations over a 31-day forecast period. The Monte Carlo simulations provide a probabilistic framework for predicting the price trajectories of Bitcoin, taking into account the two key features of its behavior, which are sudden price changes and volatility clustering.

The outcomes of the simulation demonstrate that the Bates model is capable of accurately capturing the unpredictable price fluctuations of Bitcoin and providing insightful information for short- to medium-term forecasting in extremely turbulent markets. By representing possible future outcomes, each simulated path enables market participants to see various situations and evaluate the risks. The Bates model offers a practical basis for risk assessment and decision-making in uncertain market conditions by simulating both continuous variance and jump-induced price volatility.

The applicability and robustness of the Bates model might be further improved in a number of areas for future study. First, a sensitivity analysis of important parameters—especially those controlling the jump components—may offer insight into how well the model performs and how sensitive it is to changing circumstances. This would enable focused improvements and assist in determining which parameter changes have the most effects on prediction accuracy. Furthermore, even if the model performed well during the sample period, extending out-of-sample testing to other time periods and asset classes might confirm that it is flexible enough to adjust to shifting market conditions. By demonstrating when the combined structure of volatility and jumps is most advantageous, comparative studies employing simpler models, such as pure stochastic volatility or jump-diffusion models, could assist in emphasizing the special benefits of the Bates model.

Another useful factor to take into account is computational efficiency. Despite their value, Monte Carlo simulations are computationally demanding, particularly for high-frequency or real-time applications. To make the model more practical for real-time application, future studies might investigate optimization strategies like variance reduction or parallel processing.

Finally, because the Bates model depends on certain assumptions, such mean-reverting variance and Poisson-distributed jumps, investigating different assumptions or hybrid models might increase the model's adaptability even further. The model would function better throughout extended periods of high or low volatility with adjustments that account for diverse market behaviors, increasing its ability to adapt in various kinds of financial conditions.

Despite the promising results, this study is not without limitations. First, the model calibration relies on a single year of historical Bitcoin data, which may not fully capture the range of structural changes, regulatory shocks, or macroeconomic shifts that affect long-term behavior. Additionally, the use of fixed jump and volatility parameters across the simulation horizon assumes market conditions remain constant, which may oversimplify the evolving nature of cryptocurrency markets. While the 95% confidence interval provides insight into forecast uncertainty, it does not guarantee coverage of all extreme events, particularly black swan events

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that deviate significantly from historical patterns. Lastly, the current framework does not incorporate market microstructure features such as order book dynamics, liquidity constraints, or investor sentiment, which could further enhance the realism and precision of Bitcoin price modeling in future work.

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