

Developing problem-based learning student worksheet on matrix materials to improve students' critical thinking skills

Zahro Khoirunisa, Burhanudin Arif Nurnugroho*

Universitas Ahmad Dahlan, Jl. Ahmad Yani, Tamanan, Banguntapan, Bantul, DIY 55191, Indonesia

*Corresponding e-mail: burhanudin@pmat.uad.ac.id

ARTICLE INFO

ABSTRACT

Article History

Received 6 August 2024

Revised 26 October 2024

Accepted 12 November 2024

Keywords

Critical thinking

Matrix

Problem-based learning

Worksheet

How to cite this article:

Khoirunisa, Z., & Nurnugroho, B. A. (2024). Developing problem-based learning student worksheet on matrix materials to improve students' critical thinking skills. *Bulletin of Applied Mathematics and Mathematics Education*, 4(2), 95-116.

This research was conducted based on problems at MAS Taruna Al Quran Yogyakarta, that the mathematics learning still relied on a publisher generated sources which only focused on students' understanding, but not yet targeted their middle-low critical thinking skills. It then aims to develop a problem-based learning worksheet on matrix material, which can improve students' critical thinking skills, especially on interpretation and inference indicators. The research used ADDIE development model which has five stages, including analysis, design, development, implementation, and evaluation. Finally, it successfully developed the worksheet rich of learning activities stimulating the students' critical thinking through the syntax of problem-based learning. The media expert assessment scored 46/50 (very good) and the content expert assessment scored 100/105 (very good). The worksheet also considered practical for use, as shown by the average results of participant questionnaire response scored 84.4 (very good) in the small-scale test and scored 90.5 (very good) in the large-scale test. The developed worksheet also created a significant influence in improving the students' critical thinking skills by an increase of 25.3%.

This is an open access article under the [CC-BY-SA](https://creativecommons.org/licenses/by-sa/4.0/) license.



Introduction

According to Yadav (2017), mathematics is the scientific study of quantities, including the relationships between operations and measurements which are characterized by numbers and symbols. Herdiansyah (2018) also suggests that mathematics subjects are considered difficult for most students to understand compared to other subjects. Students often experience difficulty in applying mathematics contextually. Therefore, to overcome the difficulties experienced by students, skills are needed. According to Agus and Purnama (2022), critical thinking skills are one of the important skills that students have in the mathematics learning process.

Firdaus and Wilujeng (2018) explain that critical thinking skills are the ability to analyze and evaluate students in relating information and material obtained during the learning process. Glaser (in Kurniawati & Ekayanti, 2020) also elaborates that critical thinking skills are applied in examination methods and logical reasoning. The indicator of critical thinking, according to Pertiwi

(2018) is that students can interpretation, analysis, evaluation and inference from contextual problems. It is important for students to have critical thinking skills so that students can make decisions or take action on the problems they encounter.

The development of interesting, creative and innovative teaching materials can create a learning atmosphere that is fun, encouraging and causes students to think critically used to encourage students' active role in the learning (Zuriah et al., 2016). According to Amalia Yunia Rahmawati (2020), worksheet is a instrument to help and facilitate the formation of effective interactions between students and teachers, and can improve students' activities and learning outcomes during the learning process. According to Rosliana (2019), the learning process using worksheet will provide opportunities for students to construct conceptual understanding and students can play an active role in the learning process so that they can familiarize students with critical thinking skills. To reach the objective, the use of worksheet also requires a learning model.

According to Agustina et al. (2018), problem-based learning (PBL) is a learning model that supports the active role of students and can improve critical thinking skills because learning with this model students are responsible and independent in finding solutions to problems given and can provide conclusions from solutions to these problems in their own language. According to Yulianti et al. (2022), the increase in critical thinking skills is due to the use of PBL worksheet during the learning process the material provided encourages the active role of students so as to reduce teacher dominance in the learning implementation process. According to Pansa (2016), the reasons why the learning process in schools uses the PBL is because: (1) it can prepare students to apply their learning to real world situations, (2) it makes students become knowledge producers, not just consumers of knowledge, (3) supports students to improve communication, reasoning, and critical thinking skills.

Based on the results of an interview with one of the mathematics teachers in class XI IPA MAS Taruna Al Quran, namely Ms. Aulia Fonda, S.Pd., M.Pd. on Thursday, October 05, 2023 and results pretest of matrix multiplication material to students on Tuesday, October 24 2023, proves that students' critical thinking skills are still moderate. Percentage value of critical thinking skills indicators from the results pretest of matrix multiplication material is presented in Figure 1.

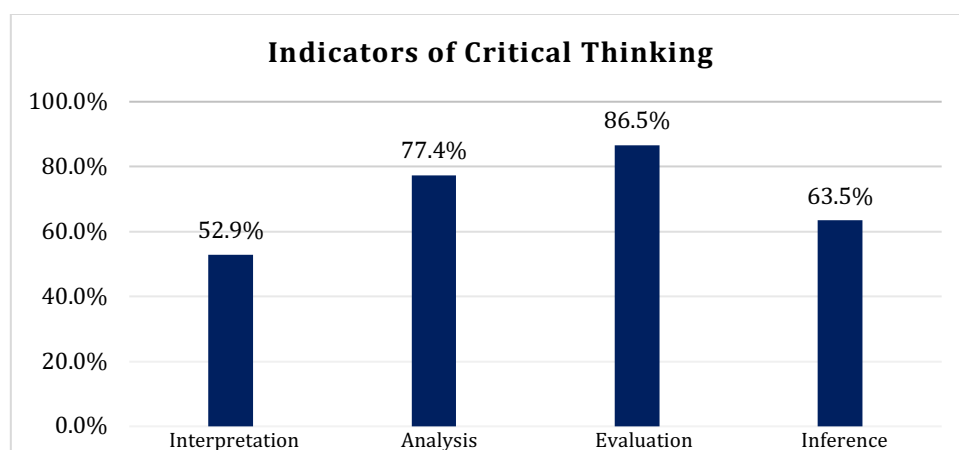


Figure 1. Pretest results on critical thinking indicators

Based on Figure 1, it shows that the average achievement of students' critical thinking skills indicators in matrix multiplication material is 70.1% in the medium category. However, the average value of students' critical thinking skills in matrix multiplication material is 70, meaning that this value is still below the minimum completeness criteria determined by the school, namely 75. This

is because the interpretation indicator obtained a percentage value of 52.9% with low category, and the inference indicator obtained a percentage value of 63.5% in the medium category, so this study will focus on improving interpretation and inference indicators.

This is also caused by the mathematics learning process using a direct learning model which tends to be one-way so that the active role of students in the learning implementation process is very minimal and the teaching materials used during the learning process are using worksheet from publishers. The worksheet presents a summary of material and questions that only focus on the students' understanding stage and does not yet have model steps PBL and critical thinking indicators. Students have not yet had the initiative to do practice questions so the teacher must give orders first, and students at the school cannot access the internet to look for other learning resources because students are not allowed to bring cellphone, laptops, and the like. Thus, the worksheet that will be developed can support students in getting new learning resources and different learning experiences by working on existing worksheet at the school, as well as making it easier for students to hone critical thinking skills.

Method

This research uses research methods Research and Development (R&D). In this research, researchers used the ADDIE development model developed by Dick and Carry and has 5 levels namely analysis, design, development, implementation, and evaluation. According to Widyastuti and Susiana (2019), the reason why the ADDIE model is still very relevant to use is because the ADDIE model has a structured framework and there is evaluation and revision at each stage, as well as a way to solve learning problems related to learning resources and student characteristics. This research intends to develop a product in the form of PBL-based worksheet on matrix material so that after using this worksheet it is hoped that it can improve students' critical thinking skills.

The subjects in this research were media expert validators, material expert validators, and class XI IPA MAS Taruna Al Quran students. The instruments used in this research are non-test instruments and test instruments. The non-test instruments are in the form of validation sheets and student response questionnaires, while the test instruments are in the form of questions pretest and posttest. In this research, there are two types of data, namely qualitative data obtained from interviews, suggestions from validators and students, and quantitative data obtained from the results of questionnaire assessments by validators and students and student test results.

Data analysis was carried out to obtain information that the worksheet being developed was said to be valid and practical. The assessment rules for worksheet products are developed using the Likert scale in Table 1 (Effendi et al., 2021).

Table 1. Validity and practicality assessment guidelines

Responses	Score
Strongly agree	5
Agree	4
Simply Agree	3
Don't agree	2
Strongly Disagree	1

From calculating the score of each statement, the average is found using the formula:

$$\bar{X} = \frac{\sum_{k=1}^n X_k}{n} \quad (1)$$

with:

\bar{X} : average worksheet assessment score by validators

X_k : score of the 2nd indicator statement k, Where k=1, 2, 3, ..., n

n : number of assessment items.

According to Habsyi et al. (2022), the criteria for the validity and practicality of the worksheet can be seen in Table 2.

Table 2. Criteria for validity and practicality of worksheet

Score Range	Category
$\bar{X} > \bar{X}_i + 1.80 SB_i$	Very Good
$\bar{X}_i + 0.60 SB_i < \bar{X} \leq \bar{X}_i + 1.80 SB_i$	Good
$\bar{X}_i - 0.60 SB_i < \bar{X} \leq \bar{X}_i + 0.60 SB_i$	Less Enough
$\bar{X}_i - 1.80 SB_i < \bar{X} \leq \bar{X}_i - 0.60 SB_i$	Not Enough
$\bar{X} \leq \bar{X}_i - 1.80 SB_i$	Poor

Information:

\bar{X} : average score

\bar{X}_i : ideal average score

$\bar{X}_i = \frac{1}{2} \times (\text{ideal maximum score} + \text{ideal minimum score})$

SB_i : ideal standard deviation

$SB_i = \frac{1}{6} \times (\text{ideal msximum score} - \text{ideal minimum score})$

Ideal maximum score = number of criteria items \times highest score

Ideal minimum score = number of criteria items \times lowest score

The worksheet developed is said to be valid and practical if it obtains a minimum average in the good category. The percentage value of critical thinking indicators can be calculated in the following way:

$$\text{Percentage Value (NP)} = \frac{\text{Obtained Score}}{\text{Maximum Score}} \times 100\% \quad (2)$$

According to Setyowati (in Karim & Normaya, 2015) the categories of critical thinking indicators can be seen in Table 3.

Table 3. Criteria of critical thinking indicator

Percentage Value	Category
$81,25 < NP \leq 100$	Very High
$71,5 < NP \leq 81,25$	High
$62,5 < NP \leq 71,5$	Currently
$43,75 < NP \leq 62,5$	Low
$0 < NP \leq 43,75$	Very Low

Results and discussion

The results of the development research that has been carried out at MAS Taruna Al Quran are printed teaching materials in the form of PBL-based worksheet.

Analysis

The analysis stage is carried out to obtain information related to initial conditions and initial product development planning. At the analysis stage, researchers carried out a needs analysis at MAS Taruna Al Quran, especially in the implementation of matrix material learning. At this analysis stage, interviews were carried out with the mathematics subject teacher, namely Mrs. Aulia Fonda, S.Pd., M.Pd. and giving pretest to students. The results of this analysis serve as guidelines for preparing PBL-based worksheet.

Curriculum and material analysis

The curriculum used at MAS Taruna Al Quran is the 2013 curriculum. At this analysis stage it focuses on the analysis of core competencies and basic competencies which will be used to formulate indicators of competency achievement as a basis for determining learning objectives for matrix material which is guided by the 2013 curriculum. will later be developed by researchers. The core competencies and basic competencies used are presented in Table 4.

Table 4. Core competencies and basic competencies

Core Competence	Basic Competence
<i>Knowledge domain</i>	
Understand, apply, analyze factual, conceptual and procedural knowledge based on curiosity about science, technology, arts, culture and humanities with insight into humanity, nationality, statehood and civilization, related to phenomena and events, as well as applying procedural knowledge in the field specific studies according to their talents and interests in solving problems.	3.3 Explain matrices, matrix similarities related to problems in everyday life and perform matrix operations including addition, subtraction, scalar multiplication, and multiplication of two matrices, as well as matrix transpose. 3.4 Analyze the properties of the determinant and inverse of a matrix of order 2×2 and 3×3 .
<i>Skills domain</i>	
Processing, reasoning, presenting in the concrete and abstract domains are related to the development of what they learn at school independently, and are able to use methods in accordance with scientific principles.	4.3 Solve problems in everyday life related to matrices and their operations. 4.4 Solving problems in everyday life related to the determinant and inverse of order 2×2 and 3×3 .

After analyzing the curriculum and materials, the researchers also conducted interviews with the mathematics teacher in class Based on the results of interviews with mathematics teachers, the matrix material includes the concept of matrices, matrix operations, and the properties of determinants and inverses of matrices. At MAS Taruna Al Quran itself, the time allocation used in the mathematics learning process per week is 4×35 minutes or two meetings. Based on the time allocation, the matrix material is taught in 4 meetings (8×35 menit) at MAS Taruna Al Quran. The worksheet developed will present problems in everyday life related to matrix material so that student can solve these problems using problem-based learning syntax.

Analysis of situations and conditions

Researchers conducted an analysis of situations and conditions related to learning models and teaching materials used by teachers during the learning process. The learning model used by the

teacher is a direct learning model with a lecture method which tends to be one way, listening to explanations from the teacher so that student involvement in the learning process is very minimal and the teaching materials used during the matrix learning process use worksheet from the publisher. The worksheet presents a summary of material and questions that only focus on the students' understanding stage and does not yet have model steps Problem Based Learning and have not been able to improve students' critical thinking skills. Students at the school also cannot access the internet to look for other learning resources because students are not allowed to bring them cellphone, laptops, and the like. So that in the learning process, teaching materials are needed that are able to motivate students to be more actively involved, so the researchers developed a work-based worksheet Problem Based Learning which has steps in the learning process starting from problem orientation, organizing students, guiding investigations, developing and presenting problems, and analyzing and evaluating. The developed worksheet also contains indicators of critical thinking including interpretation, analysis, evaluation and inference.

Analysis of student characteristics

Researchers conduct an analysis of students who will be research subjects to obtain information related to critical thinking skills which contain indicators of student interpretation, analysis, evaluation and inference in the matrix material. Researchers analyzed students' critical thinking skills, especially in matrix multiplication material which still left problems according to the results pretest on Tuesday, October 24 2023, namely the interpretation indicator is still in the low category because students are not used to writing down what they know and what is asked about the problems presented. Researchers also need to pay attention to the inference indicators because even though the inference indicators are included in the medium category, only 1 out of 26 students have achieved the inference indicators.

Based on the analysis of these students, the researcher developed a student-based worksheet Problem Based Learning to improve students' critical thinking skills.

Design

Several things that must be done in the product planning stage of worksheet development are as follows.

Material assessment

Based on the analysis stage, the material used to develop the WORKSHEET is matrix material for the odd semester of class XI SMA/MA equivalent. Then determine indicators of competency achievement and learning objectives from the matrix material in accordance with core competencies and basic competencies based on the 2013 curriculum. Matrix learning in this worksheet uses a learning model PBL and in accordance with critical thinking indicators.

Early product design

Printed teaching materials in the form of PBL-based worksheet uses A4 paper size (21 cm x 29.5 cm), using Roca One writing type with size 12. Preparing the worksheet begins with making a design cover foreword, foreword, table of contents, list of syntax Problem Based Learning and critical thinking indicators, concept maps, and worksheet contents pages. This worksheet consists of four activities, namely Worksheet I discussing matrix concepts, Worksheet II discussing addition and subtraction of two matrices, Worksheet III discussing matrix multiplication, and Worksheet IV discussing determinants and inverses of matrices.

Making research instruments

The research instrument used was a questionnaire prepared to evaluate the worksheet that had been created. Making the instrument begins with preparing a grid according to the aspects that have been determined for each questionnaire. This research instrument consists of a media expert validation sheet, material expert validation sheet, student response questionnaire, and test instruments. The four research instruments must be validated by expert validators from Mathematics Education lecturers at Ahmad Dahlan University, namely Mr. Fariz Setyawan, S.Pd., M.Pd. first before using it to assess the worksheet product that has been developed. The validation sheet that has been created according to the grid is given to media expert validators and material expert validators to test the feasibility and quality of the worksheet that has been created before being tested by students. Then, student response questionnaires were given to students after the worksheet was made suitable for testing in small and large classes. Meanwhile, the test instrument consists of: pretest and posttest used to determine the level of students' critical thinking skills.

Development

This stage is a process where everything that is needed or that supports the worksheet development process must be prepared.

Preparation or creation of worksheet

The author prepares or makes the worksheet according to the design results, makes pictures, types and colors the worksheet which is developed with the help of Canva. The results of preparing or making worksheet are as follows.

1. Cover front, contains the title of the worksheet, an image illustrating the matrix material, and the name of the compiler (See Figure 2).

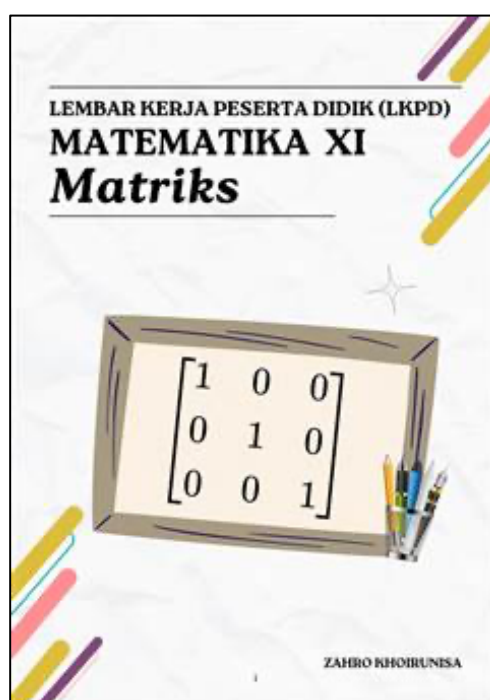


Figure 2. Cover design

- The foreword contains expressions of gratitude and thanks, a general description of the contents of the worksheet, and the author's hopes in compiling the worksheet.
- Table of contents contains information on the content page that the reader will go to.
- Syntax hints Problem Based Learning and critical thinking indicators, containing symbols or illustrations in each syntax Problem Based Learning and indicators of critical thinking.
- Concept map, contains a general description of the material to be studied in the worksheet.
- The content page (See Figure 3) of the worksheet contains basic competencies, indicators of competency achievement, learning objectives, instructions for working on the worksheet, student identity, and pearls of wisdom about the virtues of studying, taken from the Al-Quran and hadith as material to motivate students to study. The contents page of this worksheet also contains questions and steps for solving them according to the syntax Problem Based Learning which begins with student orientation, organizing students, guiding investigations, developing and presenting problems, analyzing and evaluating, and the steps for solving them also according to critical thinking indicators including interpreting, analyzing, evaluating and inferring.



Figure 3. Worksheet content page design

Product validation

Product validation in the form of PBL-based worksheet carried out after making the worksheet by the writer who is guided by the supervisor. Product validation is carried out to test the suitability of the media and materials in the worksheet that has been created.

Media expert validation

Media expert validation was carried out by 2 experts, namely Mr. Fariz Setyawan, S.Pd., M.Pd. as a lecturer in the Mathematics Education study program at Ahmad Dahlan University and Mrs. Aulia Fonda, S.Pd., M.Pd. as a mathematics subject teacher at MAS Taruna Al Quran by filling in validation sheets for each aspect of the media expert validation assessment, namely appearance, letters and images, language, and physical criteria. The assessment data by media expert validators is presented in Table 5.

Based on Table 5, the details of the average assessment from media expert validators for each aspect of the assessment are obtained, namely in the appearance aspect, the average is 5 in the very

good category, in the letter and image aspect, the average is 28 in the very good category, in the aspect language obtained an average of 8.5 in the very good category, and in the physical criteria aspect obtained an average of 4.5 in the very good category, so that the total number and total average obtained from the assessment of material expert validators were consecutive. are 92 and 46 in the very good category.

Table 5. Media expert validation results

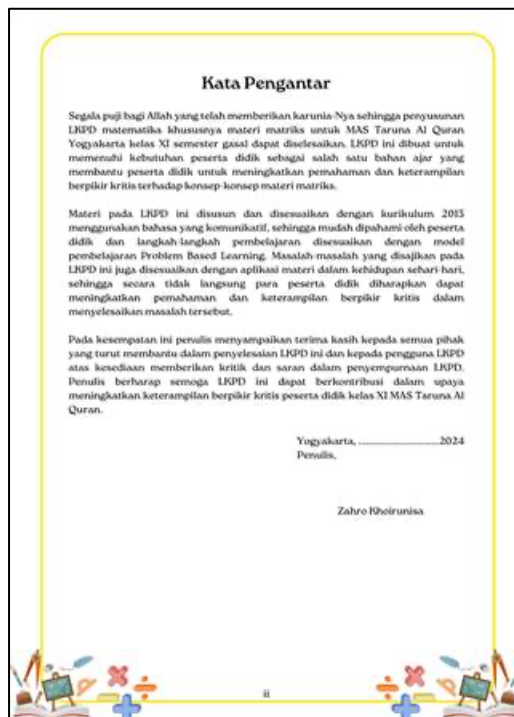
No	Assessment Aspects	Validator		Average	Category
		1	2		
1	Appearance	5	5	5	Very Good
2	Letters and	27	29	28	Very Good
3	Language	9	8	8.5	Very Good
4	Physical Criteria	4	5	4.5	Very Good
	Final score			46	Very Good

The worksheet media that was developed received this category also through several suggestions from media expert validators to be improved so that the worksheet developed was better and suitable for use can be seen in Table 6.

Table 6. Media expert validation suggestions and improvements

Product suggestions and improvements

Changed all font types on the worksheet from Roca One font to Calibri type.



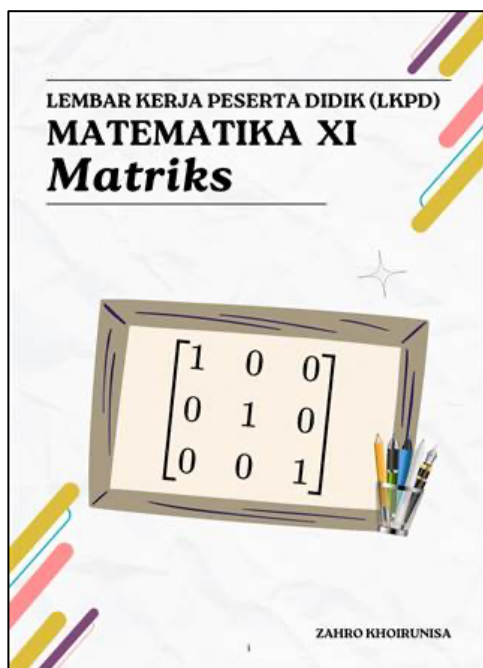
before revision



after revision

Product suggestions and improvements

Adding the university logo to the page cover.



before revision



after revision

Changed the design of the table of contents to make it less formal.

Daftar Isi	
Halaman Sampul	i
Kata Pengantar	ii
Daftar Isi	iii
Sintaks Problem Based Learning	1
Indikator Berpikir Kritis	1
Peta Konsep	2
Lembar Kerja Peserta Didik 1	3
Lembar Kerja Peserta Didik 2	7
Lembar Kerja Peserta Didik 3	11
Lembar Kerja Peserta Didik 4	19

before revision

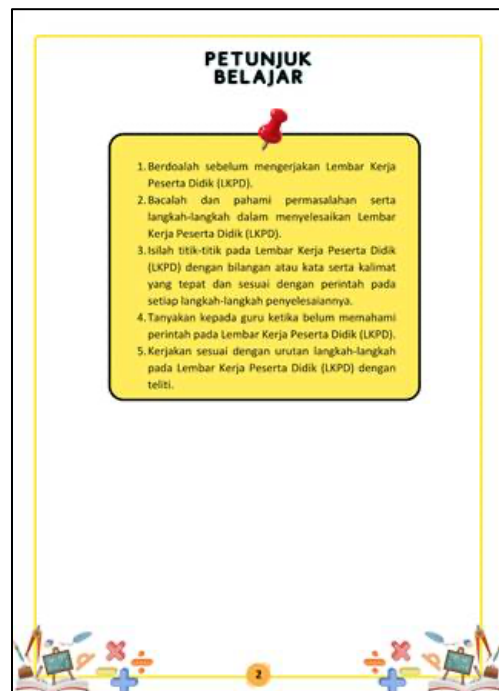
DAFTAR ISI	
HALAMAN SAMPL	i
KATA PENGANTAR	ii
DAFTAR ISI	iii
DAFTAR TABEL	iv
DAFTAR GAMBAR	v
SINTAKS PROBLEM BASED LEARNING	1
INDIKATOR BERPIKIR KRITIS	1
PETUNJUK	2
PETA KONSEP	3
LEMBAR KERJA PESERTA DIDIK 1	4
LEMBAR KERJA PESERTA DIDIK 2	8
LEMBAR KERJA PESERTA DIDIK 3	12
LEMBAR KERJA PESERTA DIDIK 4	20
DAFTAR PUSTAKA	28
BIOGRAFI PENYUSUN	29

after revision

Product suggestions and improvements

Added study instructions using worksheet.

Before the revision it did not include study instructions.



after revision

The position of the image in the worksheet activity is placed next to the question text and adds the source of the image.



before revision



after revision

Product suggestions and improvements

Font size and *equation* must be consistent.

Misalkan:
 x : harga
 y : harga
 z : harga

Sistem persamaan linear yang terbentuk dari permasalahan diatas adalah

Mengembangkan dan Menyajikan Masalah

a. Ubahlah sistem persamaan linear yang terbentuk pada langkah sebelumnya ke dalam bentuk matriks $AX = B$. A merupakan matriks koefisien, B merupakan matriks konstanta, dan X merupakan matriks variabel.

$$\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} Rp. \\ Rp. \\ Rp. \end{bmatrix}$$

b. Carilah determinan matriks A .

c. Carilah koefaktor matriks A .

$$\begin{matrix} k_{11} = \dots & k_{12} = \dots & k_{13} = \dots \\ k_{21} = \dots & k_{22} = \dots & k_{23} = \dots \\ k_{31} = \dots & k_{32} = \dots & k_{33} = \dots \end{matrix}$$

25

before revision

Misalkan:
 x : biaya
 y : biaya
 z : biaya

Sistem persamaan linear yang terbentuk dari permasalahan diatas adalah

Mengembangkan dan Menyajikan Masalah

1. Ubahlah sistem persamaan linear yang terbentuk pada langkah sebelumnya ke dalam bentuk matriks $PX = Q$. P merupakan matriks koefisien, Q merupakan matriks konstanta, dan X merupakan matriks variabel.

$$\begin{bmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} Rp. \\ Rp. \\ Rp. \end{bmatrix}$$

2. Carilah determinan matriks P .

3. Carilah koefaktor matriks P .

$$\begin{matrix} k_{11} = \dots & k_{12} = \dots & k_{13} = \dots \\ k_{21} = \dots & k_{22} = \dots & k_{23} = \dots \\ k_{31} = \dots & k_{32} = \dots & k_{33} = \dots \end{matrix}$$

26

after revision

Added points and places to answer commands at each step of completing the worksheet.

Membimbing Penyelidikan

Sajikan informasi yang diperoleh pada tabel berikut

Tabel I

Rak Buku		

Tabel II

Perampungan		

Mengembangkan dan Menyajikan Masalah

a. Sajikan bilangan-bilangan pada tabel I dan tabel II sesuai dengan letak baris dan kolom dengan mengisikan judul baris dan kolom, serta bilangan-bilangan tersebut diberi tanda kurung (tabel I dinotasikan dengan dan tabel II dinotasikan dengan), sehingga diperoleh:

17

before revision

Membimbing Penyelidikan

Sajikan informasi yang diperoleh pada langkah II1 pada tabel berikut

Tabel VIII

	Pemotongan	Perampungan
Rak Buku
Meja

Tabel IX

	Yogyakarta	Surabaya
Pemotongan
Perampungan

Mengembangkan dan Menyajikan Masalah

1. Sajikan bilangan-bilangan pada tabel VII dan tabel IX sesuai dengan letak baris dan kolom dengan mengisikan judul baris dan kolom, serta bilangan-bilangan tersebut diberi tanda kurung (tabel VIII dinotasikan dengan dan tabel IX dinotasikan dengan), sehingga diperoleh:

18

after revision

Material expert validation

Material expert validation was carried out by 2 experts, namely Mrs. Dian Ariesta Y., M.Pd. as a lecturer in the Mathematics Education study program at Ahmad Dahlan University and Mrs. Aulia Fonda, S.Pd., M.Pd. as a mathematics subject teacher at MAS Taruna Al Quran by filling out validation sheets for each aspect of the material expert validation assessment, namely appropriateness of content, indicators of critical thinking, language, appropriateness of presentation, and syntax Problem Based Learning. The assessment data by media expert validators is presented in Table 7.

Table 7. Material Expert Validation Results

No	Assessment aspects	Validator		Average	Category
		1	2		
1	Content Qualification	21	25	23	Very Good
2	Critical Thinking	16	16	16	Good
3	Language	20	21	20.5	Good
4	Feasibility of	16	20	18	Very Good
5	PBL Syntax	20	25	22.5	Very Good
Final score				100	

Based on Table 6, the details of the average assessment from material expert validators for each aspect of the assessment are that in the aspect of content suitability, the average was 23 in the very good category, in the critical thinking indicator aspect, the average was 16 in the good category, in the aspect linguistics got an average of 20.5 in the good category, in the aspect of appropriateness of presentation it got an average of 18 in the very good category, and in the aspect of syntax Problem Based Learning obtained an average of 22.5 in the very good category, so that the total number and total average obtained from the material expert validator assessment were respectively 200 and 100 in the very good category. The worksheet material developed received this category also through several suggestions from material expert validators to be improved so that the worksheet developed was better and suitable for use can be seen in Table 8.

Implementation

At this stage, the results of the worksheet that have been developed and validated by the validator will be carried out in the implementation or trial phase. At this stage, two stages of testing will be carried out as follows.

Small-scale trials

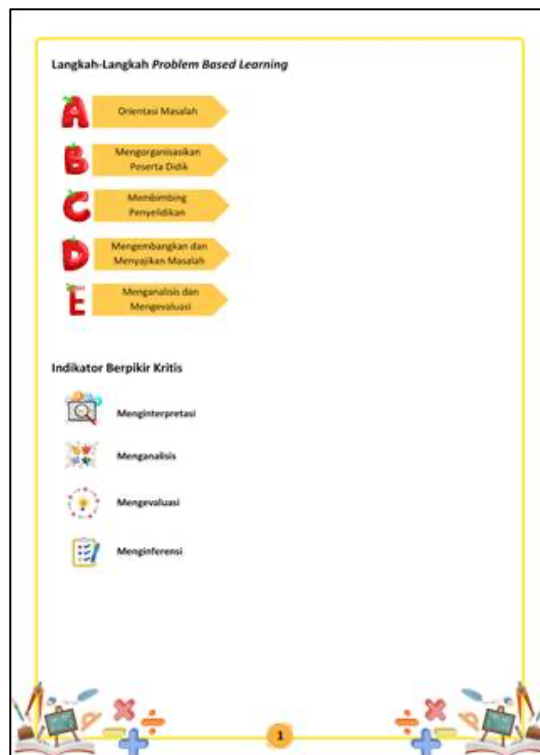
This small-scale trial was carried out by 5 students of class XI IPA MAS Taruna Al Quran. This small-scale trial was carried out on April 30 2024 by means of students investigating, studying, studying, using the worksheet, and assessing the worksheet that had been created (See Figure 4). Then students fill out a student response questionnaire to obtain information about the shortcomings of the worksheet that has been created. The results of the student response questionnaire assessment show that the worksheet developed is included in the very good category.

Table 8. Media expert validation suggestions and improvements
Suggestions and improvements

The numbers in the table of contents are ambiguous in syntax *Problem Based Learning*, so the researcher changed the symbols in the syntax *Problem Based Learning*.



before revision



after revision

Adding the concept of matrix order to the Worksheet I.

Perhatikan matriks A yang terbentuk dari Tabel D dan matriks yang terbentuk dari Tabel E)
Apakah matriks A = matriks ? Berikan alasannya

Sehingga dapat dimengerti bahwa kesamaan dua matriks atau dua matriks dikatakan sama apabila pada matriks pertama sama dengan pada matriks kedua.

Perhatikan matriks (yang terbentuk dari Tabel E)
Jika pengertian Transpose matriks adalah matriks baru yang diperoleh dari matriks sebelumnya dengan menukar elemen baris menjadi elemen kolom dan elemen kolom menjadi elemen baris. Misal matriks A ditransposisikan maka dapat dinotasikan dengan A^T sehingga jika matriks (yang terbentuk dari Tabel E) ditransposisikan maka dapat dinotasikan dengan dan diperoleh matriks berikut:

$$T = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

Menganalisis dan Mengevaluasi

Dari langkah-langkah sebelumnya maka dapat diperoleh pengertian

- Matriks adalah
- Kesamaan dua matriks adalah
- Transpose matriks adalah

before revision

Perhatikan matriks H (yang terbentuk dari Tabel I) dan matriks (yang terbentuk dari Tabel II)
Apakah matriks H = matriks ? Berikan alasannya

Sehingga dapat dimengerti bahwa kesamaan dua matriks atau dua matriks dikatakan sama apabila pada matriks pertama sama dengan pada matriks kedua.

Perhatikan matriks (yang terbentuk dari Tabel II)
Jika pengertian Transpose matriks adalah matriks baru yang diperoleh dari matriks sebelumnya dengan menukar elemen baris menjadi elemen kolom dan elemen kolom menjadi elemen baris. Misal matriks H ditransposisikan maka dapat dinotasikan dengan H^T sehingga jika matriks (yang terbentuk dari Tabel II) ditransposisikan maka dapat dinotasikan dengan dan diperoleh matriks berikut:

$$T = \begin{bmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix}$$

Menganalisis dan Mengevaluasi

Dari langkah-langkah B sampai D maka dapat diperoleh pengertian

- Matriks adalah
- Kesamaan dua matriks adalah
- Transpose matriks adalah
- Ordo matriks adalah

after revision

Suggestions and improvements

Clarify command sentences in the syntax of developing and presenting Worksheet II and Worksheet III problems.

Mengembangkan dan Menyajikan Masalah

a. Sajikan bilangan-bilangan pada tabel I dan II sesuai dengan letak baris dan kolom dengan mengabaikan judul baris dan kolom, serta bilangan-bilangan tersebut diberi tanda kurung (tabel produksi sepatu di Yogyakarta dinotasikan dengan M dan tabel produksi sepatu di Solo dinotasikan dengan N), sehingga diperoleh:

$$M = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \quad N = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

b. Pada langkah a sudah diketahui apa yang ditanyakan pada permasalahan tersebut sehingga dapat diperoleh:

$$M \square N = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \dots \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

$$M \square N = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

dan

$$M \square N = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \dots \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

$$M \square N = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

Menganalisis dan Mengevaluasi

Dari perhitungan diatas dapat diperoleh

10

before revision

2. Apa saja yang ditanyakan dari permasalahan langkah A?

Membimbing Penyelidikan

Sajikan informasi pada langkah B1 dalam bentuk tabel berikut

Tabel II: Produksi Sepatu di Yogyakarta

	Standard	Premium
Sneakers	—	—
Boots	—	—

Tabel IV: Produksi Sepatu di Solo

	Standard	Premium
Sneakers	—	—
Boots	—	100

Mengembangkan dan Menyajikan Masalah

1. Sajikan bilangan-bilangan pada Tabel II dan Tabel IV sesuai dengan letak baris dan kolom dengan mengabaikan judul baris dan kolom, serta bilangan-bilangan tersebut diberi tanda kurung (Tabel II dinotasikan dengan M dan tabel IV dinotasikan dengan N), sehingga diperoleh:

$$M = \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix} \quad N = \begin{bmatrix} \dots & \dots \\ \dots & 100 \end{bmatrix}$$

2. Pada langkah B2 akan diketahui apa yang ditanyakan pada permasalahan tersebut sehingga dapat diperoleh:

$$M + N = \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix} \dots \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}$$

10

after revision

Provide a space in the sentence "2 kg of oranges" problem 2 Worksheet III.

Masalah 2

Orientasi Masalah

Di sebuah toko buah-buahan terdapat 3 orang pembeli yang masing-masing membeli apel, jeruk, dan anggur. Pembeli A membeli 1 kg apel, 2 kg jeruk, dan 1 kg anggur. Pembeli B 1 kg apel, 1 kg jeruk, dan 2 kg anggur. Pembeli C membeli 2 kg apel, 1 kg jeruk, dan 1 kg anggur. Jika harga 1 kg apel adalah Rp 55.000, harga 1 kg jeruk adalah Rp 25.000, dan harga 1 kg anggur adalah Rp 40.000, maka berapa uang yang harus dibayarkan oleh masing-masing pembeli?

Penyelesaian

Mengorganisasikan Peserta Didik

Informasi yang diperoleh dari masalah diatas
Diketahui: Ditanya:

Membimbing Penyelidikan

b. Sajikan informasi yang diperoleh pada tabel berikut

Tabel I

	Apel	Jeruk	Anggur
A			
B			
C			

Tabel II

	Harga
Apel	
Jeruk	
Anggur	

14

before revision

Masalah 2

Orientasi Masalah

Gambar 4

Di sebuah toko buah-buahan terdapat 3 orang pembeli yang masing-masing membeli apel, jeruk, dan anggur. Pembeli X membeli 1 kg apel, 2 kg jeruk, dan 1 kg anggur. Pembeli Y 1 kg apel, 1 kg jeruk, dan 2 kg anggur. Pembeli Z membeli 2 kg apel, 1 kg jeruk, dan 1 kg anggur. Jika harga 1 kg apel adalah Rp 55.000, harga 1 kg jeruk adalah Rp 25.000, dan harga 1 kg anggur adalah Rp 40.000, maka berapa uang yang harus dibayarkan oleh masing-masing pembeli?

Penyelesaian

Mengorganisasikan Peserta Didik

1. Apa yang diketahui dari permasalahan pada langkah A?

2. Apa yang ditanyakan dari permasalahan pada langkah A?

Membimbing Penyelidikan

Sajikan informasi yang diperoleh dari langkah B1 pada tabel berikut

Tabel VI

	Apel	Jeruk	Anggur
X			
Y			
Z			

Tabel VII

	Harga
Apel	
Jeruk	
Anggur	

15

after revision

Suggestions and improvements

Provide parentheses in the matrix determinant formula.

Membimbing Penyelidikan

Untuk menyelesaikan permasalahan diatas, perhatikan informasi berikut:

A. Determinan

Misalkan A adalah matriks persegi berordo 2 x 2, determinan dinyatakan dengan $\det A$ atau $|A|$, dan $\det (A)$ atau $|A|$ didefinisikan sebagai jumlah semua hasil kali elemen bertanda dari A (selisih antara perkalian elemen-elemen pada diagonal utama dengan perkalian elemen-elemen pada diagonal sekunder pada matriks A).

Berdasarkan definisi diatas maka dapat dicari nilai determinan dari matriks berikut:

$$\det A = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \begin{matrix} \text{diagonal sekunder} \\ \text{diagonal utama} \end{matrix}$$

$$|A| = a_{11} \times a_{22} - a_{12} \times a_{21}$$

Jika matriks A adalah matriks persegi, dan jika kita dapat mencari matriks B sehingga $AB = BA = I$, maka A dapat dibalik dan B dinamakan invers dari A.

Berdasarkan definisi diatas maka dapat dicari nilai invers dari matriks berikut:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}$$

Misalkan:

x : harga _____

y : harga _____

Sistem persamaan linear yang terbentuk dari permasalahan diatas adalah

$$\begin{matrix} \dots x + \dots y = 15.000 \\ \dots x + \dots y = 22.000 \end{matrix}$$

Mengembangkan dan Menyajikan Masalah

a. Ubahlah sistem persamaan linear yang terbentuk pada langkah sebelumnya ke dalam bentuk matriks $AX = B$. A merupakan matriks koefisien, B merupakan matriks konstanta, dan X merupakan matriks variabel.

$$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15.000 \\ 22.000 \end{bmatrix}$$

before revision

Membimbing Penyelidikan

Untuk menyelesaikan permasalahan diatas, perhatikan informasi berikut:

A. Determinan

Misalkan P adalah matriks persegi berordo 2 x 2, determinan dinyatakan dengan $\det P$ atau $|P|$, dan $\det (P)$ atau $|P|$ didefinisikan sebagai jumlah semua hasil kali elemen bertanda dari P (selisih antara perkalian elemen-elemen pada diagonal utama dengan perkalian elemen-elemen pada diagonal sekunder pada matriks P).

Berdasarkan definisi diatas maka dapat dicari nilai determinan dari matriks berikut:

$$\det P = |P| = \begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix} \begin{matrix} \text{diagonal sekunder} \\ \text{diagonal utama} \end{matrix}$$

$$|P| = (p_{11} \times p_{22}) - (p_{12} \times p_{21})$$

Jika matriks P adalah matriks persegi, dan jika kita dapat mencari matriks Q sehingga $PQ = QP = I$, maka P dapat dibalik dan Q dinamakan invers dari P.

Berdasarkan definisi diatas maka dapat dicari nilai invers dari matriks berikut:

$$P^{-1} = \frac{1}{|P|} \begin{bmatrix} p_{22} & -p_{12} \\ -p_{21} & p_{11} \end{bmatrix}$$

Misalkan:

x : harga _____

y : harga _____

Sistem persamaan linear yang terbentuk dari permasalahan diatas adalah

$$\begin{matrix} \dots x + \dots y = 15.000 \\ \dots x + \dots y = 22.000 \end{matrix}$$

Mengembangkan dan Menyajikan Masalah

1. Ubahlah sistem persamaan linear yang terbentuk pada langkah C ke dalam bentuk matriks $PX = Q$. P merupakan matriks koefisien, Q merupakan matriks konstanta, dan X merupakan matriks variabel.

$$\begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15.000 \\ 22.000 \end{bmatrix}$$

after revision

Added explanation regarding cofactors to Worksheet IV.

Before the revision there was no explanation about cofactors.

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

$$|P| = \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix} \begin{matrix} p_{11} & p_{12} \\ p_{21} & p_{22} \\ p_{31} & p_{32} \end{matrix}$$

$$|P| = (p_{11}p_{22}p_{33} + p_{12}p_{23}p_{31} + p_{13}p_{21}p_{32}) - (p_{12}p_{21}p_{33} + p_{11}p_{23}p_{32} + p_{13}p_{22}p_{31})$$

Invers

Jika matriks P adalah matriks persegi, dan jika kita dapat mencari matriks Q sehingga $PQ = QP = I$, maka P dapat dibalik dan Q dinamakan invers dari P.

Berdasarkan definisi diatas maka dapat dicari nilai invers dari matriks berikut:

$$P^{-1} = \frac{1}{|P|} \text{adj} P$$

Keterangan:

$\text{adj}(P)$ adalah matriks transpose dari matriks kofaktor P.

Minor dan Kofaktor

Jika P adalah matriks kuadrat, maka minor dari suatu elemen p_{ij} dari matriks P yang dinyatakan dengan M_{ij} adalah hasil determinan (anak determinan) dari matriks semula setelah dihilangkan baris ke-i dalam kolom ke-j pada matriks P dan kofaktor dari suatu elemen p_{ij} dari matriks P yang dinyatakan dengan K_{ij} adalah hasil perkalian minor dengan suatu angka yang mempunyai suatu aturan yaitu $(-1)^{i+j}$, dimana i adalah baris dan j adalah kolom.

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \rightarrow M_{11} = \begin{vmatrix} p_{22} & p_{23} \\ p_{32} & p_{33} \end{vmatrix}$$

$$M_{11} = p_{22}p_{33} - p_{23}p_{32} \quad k_{ij} = (-1)^{i+j} |M_{ij}|$$

after revision

Suggestions and improvements

Added bibliography.

Before the revision there was no bibliography.



after revision

Add cover The back contains the biodata of the worksheet compiler.

Before the revision there were none cover back.



after revision



Figure 4. Students joined the small-scale trial

Large-scale trials

This trial is intended to obtain information from students regarding whether the worksheet being developed fulfills practical or feasibility aspects that can be used in the learning process. Then students fill out a student response questionnaire to obtain information about the shortcomings of the worksheet that has been created (See Figure 5). The results of the assessment of student response questionnaires in large-scale trials show that the worksheet developed is included in the very good category.



Figure 5. Students joined the large-scale trial

Evaluation

The evaluation stage in the ADDIE model is carried out at each stage starting from the analysis stage, design stage, development stage and implementation stage. The evaluation stage is carried out in order to produce worksheet that is better and suitable for use in the learning process. At the analysis stage, information related to the learning model and teaching materials used during the learning process as well as students' critical thinking skills need to be evaluated to get the right solution. Students' critical thinking skills can be identified from the results pretest which have been

carried out before using the developed worksheet. Therefore, the solution provided by researchers is to develop teaching materials in the form of PBL-based worksheet on matrix material to improve students' critical thinking skills.

At the design stage, evaluation is carried out by consulting with the supervisor regarding the initial format design and content of the worksheet being developed. After consulting with the supervisor, the researcher then created the overall worksheet. At this stage, evaluation is also carried out during validation of assessment instruments such as media expert and material expert validation sheets as well as student response questionnaires. The assessment instrument is suitable for use with several improvements based on suggestions and comments from expert validators.

At the development stage, evaluation is carried out with initial worksheet validation tests by material expert validators and media expert validators. The results of the assessment by material expert validators and media expert validators show that the worksheet is based on Problem Based Learning on matrix material to improve students' critical thinking skills which was developed in the very good category for use with several improvements based on suggestions given by the validators.

At the implementation stage, evaluation is carried out by assessing the worksheet by participants. The results of the assessment by students show that the worksheet is based on Problem Based Learning on matrix material to improve students' critical thinking skills which is developed in the very good category for use. Thus, worksheet is based Problem Based Learning The matrix material to improve students' critical thinking skills that has been developed is valid and suitable for use as supporting teaching material for the matrix material in the mathematics learning process.

This evaluation stage is also carried out posttest to students which was carried out on May 7 2024 and aims to determine students' critical thinking skills after using the worksheet developed in the learning process based on Problem Based Learning.

PBL-based worksheet which has been developed is also effectively applied in the learning process because it can help improve students' critical thinking skills. This effectiveness can be seen from the achievement of critical thinking indicators after using PBL-based worksheet which increased from before using PBL-based worksheet which is described in Figure 8.

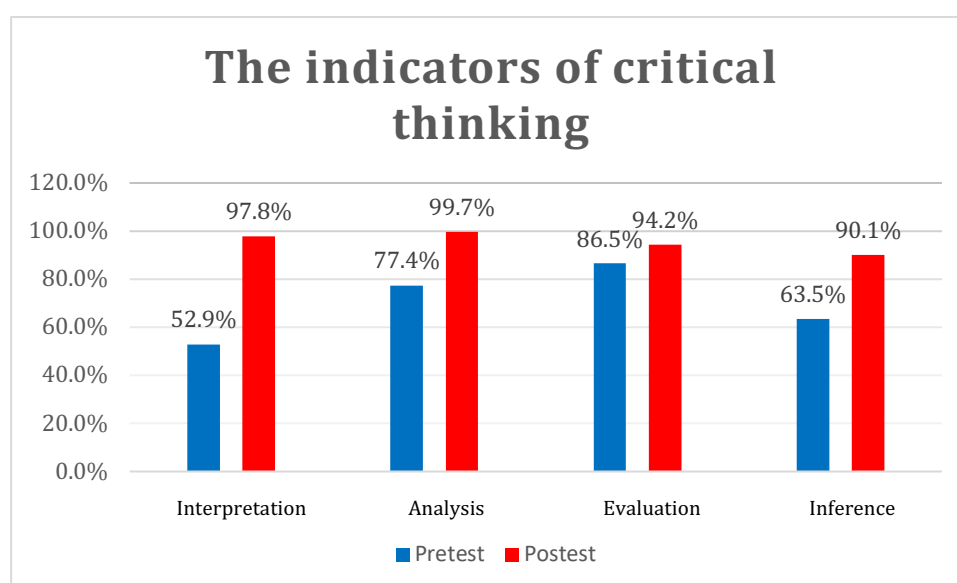


Figure 6. Results of increasing critical thinking indicators

Based on Figure 6, it can be seen that the achievement of critical thinking indicators is in the results pretest shows that interpretation indicators are in the low category with a percentage of 52.9%, analysis indicators are in the high category with a percentage of 77.4%, evaluation indicators are in the high category with a percentage of 86.5%, and inference indicators are in the medium category with a percentage of 63.5%.

Meanwhile, the achievement of critical thinking indicators after using PBL-based worksheet based on the results posttest shows that interpretation indicators are included in the high category with a percentage of 97.8%, analysis indicators are included in the high category with a percentage of 99.7%, evaluation indicators are included in the high category with a percentage of 94.2%, and inference indicators are included in the high category with a percentage of 90.1%.

Conclusion

Based on the explanation it can be concluded that:

1. Development of PBL-based worksheet on matrix material to improve students' critical thinking skills using the ADDIE development model is valid for use as shown by the average results of media expert assessments, namely 46 in the very good category and the average results of material expert assessments, namely 100 in the very good category.
2. PBL-based worksheet on matrix material that was developed was practically used, as shown by the average results of participant questionnaire response assessments in the small-scale test, namely 84.4 in the very good category and the average results of participant response questionnaire assessments in the large-scale test, namely 90.5 in the very good category.
3. The developed PBL-based worksheet on matrix material has a significant influence in improving critical thinking skills with an increase of 25.3%. This is shown from the test results paired sample t-test which obtains the value of sig (2-tailed) equal to $0.000 < 0.05$ or it can be concluded that the average test results of students' critical thinking skills after using PBL-based worksheet higher than the critical thinking skills test results before using PBL-based worksheet.

References

- Agus, I., & Purnama, A. N. (2022). Kemampuan berpikir kritis matematika siswa: Studi pada siswa SMPN Satu Atap. *Jurnal Pendidikan Matematika Raflesia*, 7(1), 65–74.
- Agustina, R. F., Soro, S., & Pradipta, T. R. (2018). Perbedaan kemampuan berpikir kritis matematis siswa antara problem based learning dan student team achievement division. *Prosiding SENAMKU*, 1, 259–267.
- Effendi, R., Herpratiwi, H., & Sutiarto, S. (2021). Pengembangan LKPD matematika berbasis problem based learning di sekolah dasar. *Jurnal Basicedu*, 5(2), 920–929. <https://doi.org/10.31004/basicedu.v5i2.846>
- Firdaus, M., & Wilujeng, I. (2018). Pengembangan LKPD inkuiri terbimbing untuk meningkatkan keterampilan berpikir kritis dan hasil belajar peserta didik. *Jurnal Inovasi Pendidikan IPA*, 4(1), 26–40. <https://doi.org/10.21831/jipi.v4i1.5574>
- Habsyi, R., R. M. Saleh, R., & Isman M. Nur. (2022). Pengembangan E-LKPD berbasis guided discovery learning untuk meningkatkan kemampuan berpikir kritis siswa. *Kognitif: Jurnal Riset HOTS Pendidikan Matematika*, 2(1), 1–18. <https://doi.org/10.51574/kognitif.v2i1.385>
- Herdiansyah, K. (2018). Pengembangan LKPD berbasis model problem based learning untuk meningkatkan kemampuan berpikir kritis. *Eksponen*, 8(1), 25–33. <https://doi.org/10.47637/eksponen.v8i1.138>
- Karim, K., & Normaya, N. (2015). Kemampuan berpikir kritis siswa dalam pembelajaran dalam

- pembelajaran matematika dengan menggunakan model Jucama di sekolah menengah pertama. *EDU-MAT: Jurnal Pendidikan Matematika*, 3(1). <https://doi.org/10.20527/edumat.v3i1.634>
- Kurniawati, D., & Ekayanti, A. (2020). Hubungan antara berpikir kritis dan pembelajaran matematika. *Jurnal Penelitian Tindakan Kelas dan Pengembangan Pembelajaran*, 3(1), 1–10.
- Pansa, H. E. (2016). Problem-based learning dalam pembelajaran matematika. *Prosiding Konferensi Nasional Penelitian Matematika Dan Pembelajarannya (KNPMP I)*, 703–712.
- Pertiwi, W. (2018). Analisis kemampuan berpikir kritis matematis peserta didik SMK pada materi matriks. *Jurnal Pendidikan Tambusai*, 2(4), 793–801.
- Rahmawati, A. Y. (2020). *Pentingnya LKPD Pada Pendekatan Scientific Pembelajaran Matematika*. 3(July), 1–23.
- Roslina, I. (2019). Pengembangan LKPD matematika dengan model learning cycle 7E berbantuan mind mapping. *Jurnal Pengembangan Pembelajaran Matematika*, 1(1), 10–22. <https://doi.org/10.14421/jppm.2019.11.10-22>
- Yadav, K. D. (2017). Exact definition of mathematics. *International Research Journal of Mathematics, Engineering and IT*, 4(1), 34–42.
- Yulianti, D., Rochmiyati, R., Pramudiyanti, P., Adha, M. M., Isnawati, I., Sabrini, A. T., & Fitriadi, F. (2022). Workshop implementasi LKPD berbasis problem based learning pada pendidik sekolah dasar. *Jurnal Sinergi*, 3(2), 54–59.
- Zuriah, N., Sunaryo, H., & Yusuf, N. (2016). IBM guru dalam pengembangan bahan ajar kreatif inovatif berbasis potensi lokal. *Dedikasi*, 13, 39.

This page is intentionally left blank.