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Distribution of prime numbers

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ABSTRACT

This research explores the distribution of prime numbers, which are a fundamental topic in number theory. The study originated from the author's fascination with mathematics and the desire to discover something novel. The research proposes that the distribution of prime numbers follows a regular pattern starting from the number 2. The author suggests that prime numbers can be obtained by dividing certain even numbers that have four factors by the number 2, resulting in prime numbers in sequential order. This hypothesis was tested and confirmed through the practical application of the proposed mathematical formula. Additionally, the study found that even numbers greater than or equal to 8, with six or more factors, produce complex numbers. Thus, this research provides two main contributions: firstly, a mathematical formula for the distribution of prime numbers, and secondly, a formula for the distribution of complex numbers. These findings have potential applications in various mathematical fields, including cryptography and problem-solving in number theory.

Introduction

Numbers are one of the most intriguing topics in mathematics. Since ancient times, they have fascinated and astonished scientists, as well as amateurs like myself, who are passionate about mathematics. Prime numbers such as 2, 3, 5, 7, 11, 13, 17, and so on, appear to be randomly distributed within the set of natural numbers, yet they exhibit certain regularities. These numbers are considered the fundamental building blocks for other numbers, similar to the role of atoms in relation to molecules. They have important applications in cryptography and data protection, and are central to the Riemann Hypothesis, one of the most famous mathematical problems of our time (Granville, 2019; Conrey, 2003).

Over the course of thousands of years, mathematicians have uncovered only a fraction of the mysteries surrounding prime numbers. Euclid, the Greek mathematician, demonstrated that there are infinitely many prime numbers. The basic idea is that if there were a finite number of prime numbers, by multiplying them together and adding one, one could produce a new number that is not divisible by any of the prime numbers in the list, thus proving there must be infinitely many prime numbers (Hardy & Wright, 2008).

In the nineteenth century, mathematicians formulated the prime number theorem, which provides an approximate estimate of the number of prime numbers less than a given number. The theorem reveals that prime numbers become less frequent as numbers grow larger, following a specific approximate mathematical formula (Tao, 2016). The study of twin primes, pairs of prime numbers with exactly one number between them, posits that there are infinitely many such pairs. Despite the conjecture's plausibility, it has neither been proven nor disproven. In a breakthrough in 2013, mathematician Yitang Zhang demonstrated that there are infinitely many pairs of prime numbers with gaps smaller than 70 million. This was a significant milestone in narrowing the constraints on prime number gaps (Zhang, 2014).

Goldbach's conjecture, another prominent study, suggests that every even number greater than 2 can be expressed as the sum of two prime numbers. While this conjecture holds for small numbers and has been verified for numbers up to 4 quintillion using modern computers and sophisticated software, it has yet to be proven for all even numbers (Tao, 2016). The investigation of palindromic primes, which read the same forwards and backwards, reveals that such primes are rare among reversible numbers regardless of the numerical system used. A notable example is Belphegor's Prime: 1000000000000066600000000000001 (Hardy & Wright, 2008).

The Riemann Hypothesis, one of the Millennium Prize Problems, extends the theory of prime numbers by providing a more precise formula that estimates the number of prime numbers less than a given number. This hypothesis, which links the zeros of a special complex function to the distribution of prime numbers, remains unproven despite substantial numerical evidence supporting it (Conrey, 2003; Bombieri, 2007).

Method

This research adopts a mathematical approach to explore the arrangement and properties of prime numbers. A prime number is defined as an integer greater than one that has no divisors other than one and itself. The initial prime numbers in the numerical series are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. In contrast, composite numbers are those with more than two natural factors, excluding the number one, which is neither prime nor composite.

Identification of Prime Numbers

To ascertain the primality of numbers, two primary methods are employed:

(1) *Factoring*; Through factorization, mathematicians can swiftly determine a number's primality. A factor is any number that, when multiplied by another, yields the original number. For instance, the prime factors of 10 are 2 and 5, as their product is 10. However, 1 and 10 are not considered prime factors

(2) *Utilization of Calculators;* Calculators aid in the division process to test for primality. For example, dividing 57 by 2 yields a non-integer result, while dividing by 3 gives 19, an integer, indicating that 57 is not a prime number as it has factors other than one and itself

(3) *Historical Context of Prime Numbers*; Prime numbers have been studied since antiquity, with Greek mathematicians like Euclid and Eratosthenes contributing significantly to their understanding. Euclid provided the first proof of the infinitude of prime numbers. The distribution of prime numbers is further elucidated by the Prime Number Theorem and the Riemann Zeta Function.

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(4) *Prime Numbers in Cryptography*: The RSA encryption system exemplifies the application of prime numbers in computing. It requires two large prime numbers for secure information transmission over the Internet. The difficulty of prime factorization underpins the security of RSA encryption.

(5) *Properties of Prime Numbers*; The research will investigate several properties of prime numbers, such as:

- Every prime number greater than 3 can be expressed in the form $6k \pm 1$ where k is a natural number.
- Every integer greater than 1 has at least one prime divisor.
- If n is composite, it has a prime divisor p less than or equal to $\sqrt{n}$.
- Prime twins are pairs of prime numbers with a difference of 2, such as $(5, 7)$ and $(11, 13)$.

(6) *Research Hypotheses*; The research hypotheses will be tested using a methodological approach that includes:

- Detailed analysis of prime number distribution.
- Application of cryptographic principles to demonstrate the practical utility of prime numbers.
- Exploration of prime number properties through computational methods.

Results and Discussion

In this results and discussion chapter, we outline empirical findings that support the hypothesis that the distribution of prime numbers can be determined through division of even numbers that have exactly four factors. Through the factor tree method, we showed that even numbers with four factors, when divided by two, yield the known prime numbers. In contrast, even numbers with more than four factors yield composite numbers. This finding not only strengthens our understanding of the nature of prime numbers but also offers a new perspective in the search for larger primes, which has important implications in the fields of cryptography and number theory.

Empirical Analysis of Prime Number Distribution

The study's empirical analysis began with the hypothesis that prime numbers can be derived from even numbers starting from 4, provided they have exactly four factors. The function for even numbers is defined as:

$$F(n) = 2n$$

Applying this function, we observe the following sequence:

$$F(1) = 2, F(2) = 4, F(3) = 6, ..$$

For odd numbers, the function is:

$$F(n) = 2n + 1$$

Yielding the sequence:

$F(1) = 3, F(2) = 5, F(3) = 7,...$

The hypothesis posits that prime numbers are the result of dividing even numbers (with exactly four factors) by 2. The equation used is:

$$P = \frac{e}{2}, \text{ where } e \geq 4 \text{ and } e \text{ has 4 factors}$$

The following results were obtained:

$$\frac{4}{2} = 2, \frac{6}{2} = 3, \frac{10}{2} = 5, \frac{14}{2} = 7, \frac{22}{2} = 11, \frac{26}{2} = 13, \frac{34}{2} = 17, \frac{38}{2} = 19, ..$$

These results align with the known sequence of prime numbers.

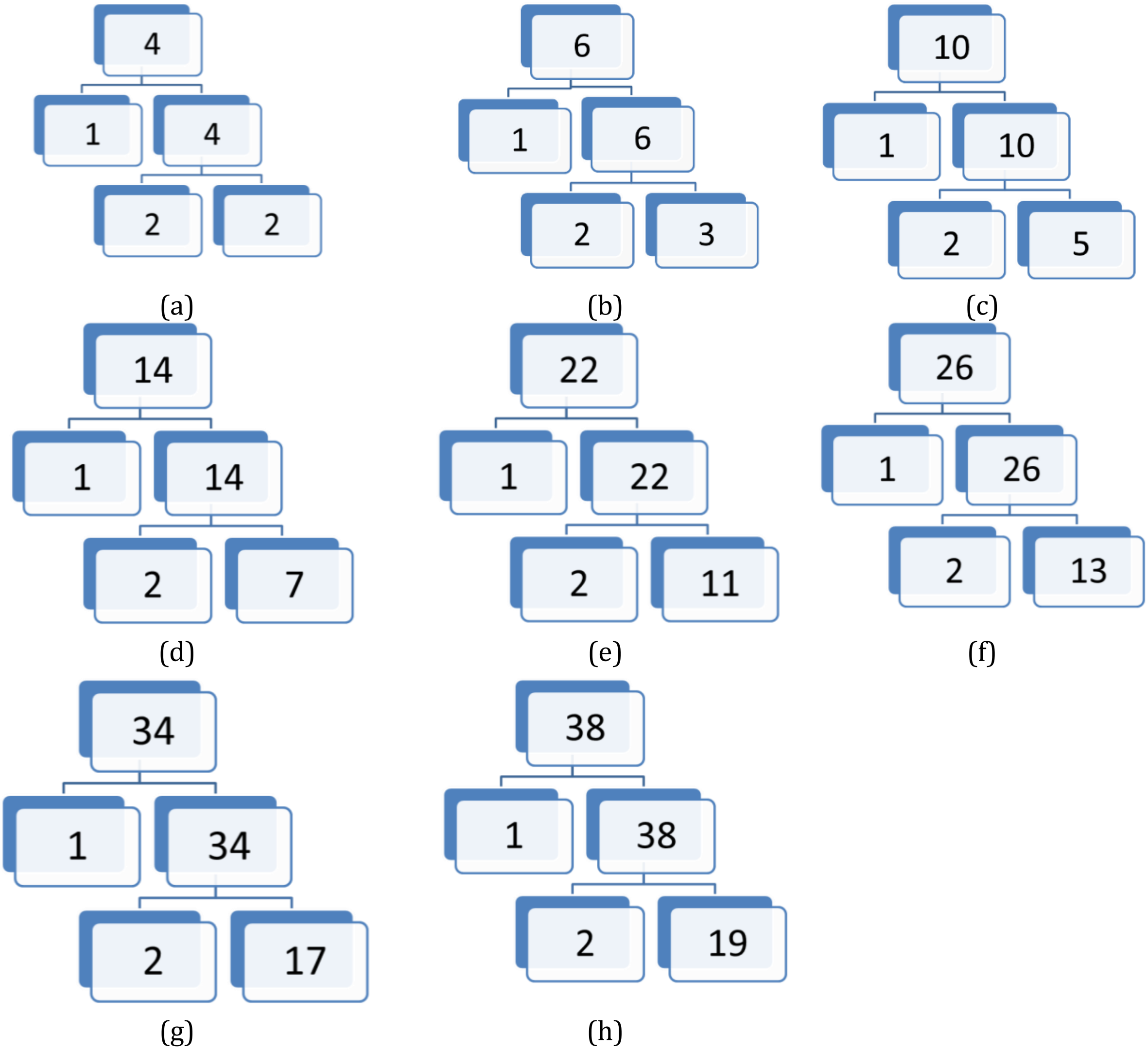

Figure 1. The first figure (a), the second figure (b), the third figure (c), the fourth figure (d), the fifth figure (e), the sixth figure (f), the seventh figure (g), and the eighth figure (h).

The figures 1, illustrate the known sequence of prime numbers. Each figure depicts the next prime number in the established sequence of prime numbers. This data provides a clear visual representation of how prime numbers evolve and are distributed among larger numbers.

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Factor Tree Method Illustration

The factor tree method was employed to illustrate the factors of the numbers in question. The method visually represents the factorization process, breaking down a composite number into its prime factors. The illustration (created using the graphic_art tool) demonstrates the factorization of even numbers with exactly four factors.

Testing the Hypothesis on Composite Numbers

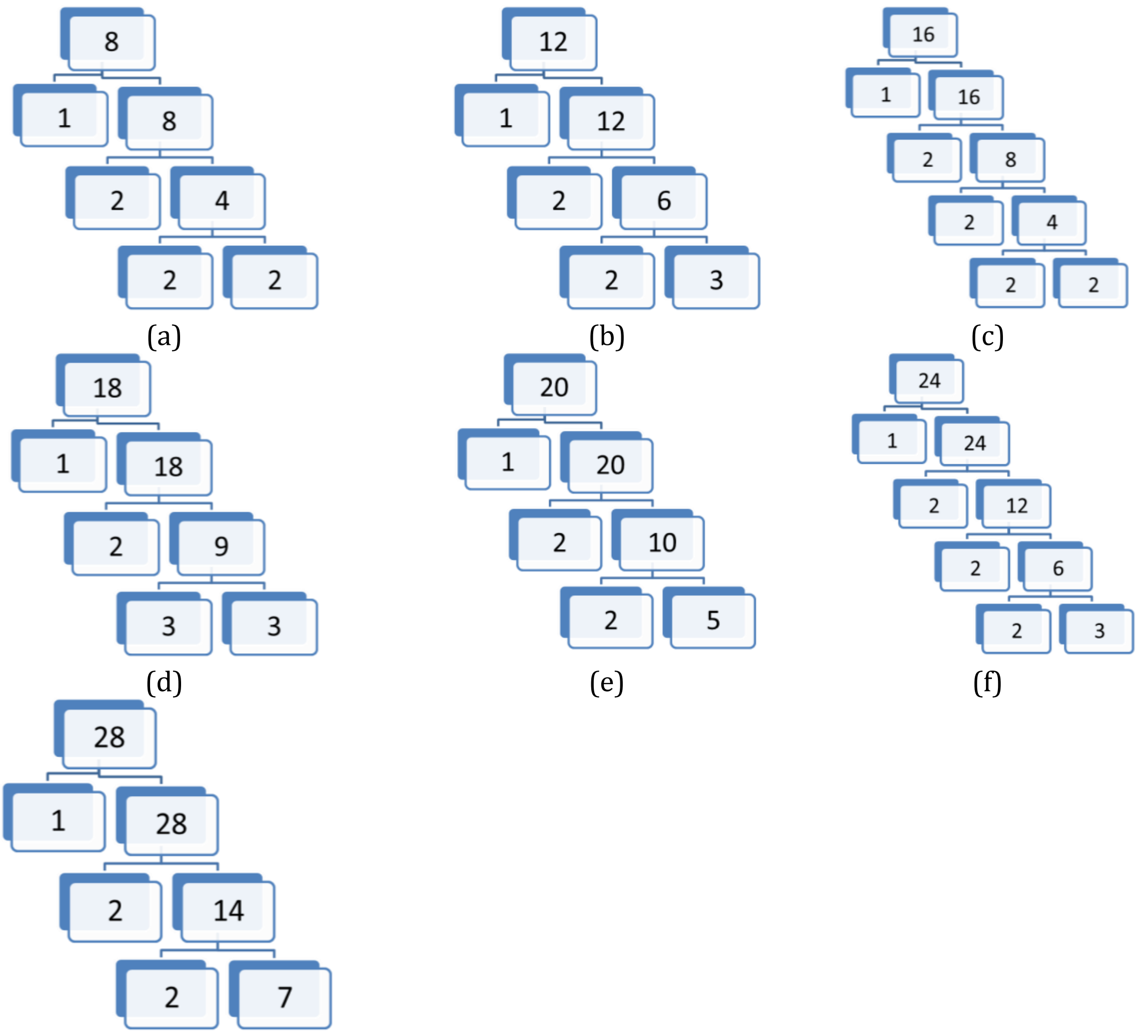
To further validate the hypothesis, even numbers equal to or greater than 8 with six or more factors were divided by 2. The equation used is:

$$c = \frac{e}{2}, \text{where } e \geq 8 \text{ and } e \text{ has 6 factors or more}$$

The results were as follows:

$$\frac{8}{2} = 4, \frac{12}{2} = 6, \frac{16}{2} = 8, \frac{18}{2} = 9, \frac{20}{2} = 10, \frac{24}{2} = 12, \frac{28}{2} = 14, \ldots$$

These results consistently yielded composite numbers, supporting the hypothesis that the distribution of composite numbers follows a sequential pattern.



(g)

Figure 2. The first figure (a), the second figure (b), the third figure (c), the fourth figure (d), the fifth figure (e), the sixth figure (f), the seventh figure (g)

Figure 2, depicted results consistently generated composite numbers, reinforcing the hypothesis that composite numbers follow a sequential pattern in their distribution. Each illustration showcases a series of composite numbers arranged in a sequential manner, indicating a discernible trend in their distribution within the numerical spectrum. This observation contributes to a deeper understanding of the underlying structure and distribution of composite numbers within the realm of mathematics.

Discussion

The findings suggest a novel approach to identifying prime numbers through the division of even numbers with specific factors. This method offers a systematic way to explore the distribution of primes and could potentially simplify the search for larger prime numbers. The factor tree method provides a clear visual representation of the factors, aiding in the understanding of the number's composition. The study's results contribute to the broader field of number theory by proposing a practical mathematical approach to prime number identification. Further research is needed to explore the implications of this method and its potential applications in cryptography and other areas of mathematics.

Conclusion

Through this research, I reached the conclusion that the distribution of prime numbers is regular according to the theoretical and practical aspects, and that is that the distribution of prime numbers is a regular order in ascending order starting from the number 2 and continuing to infinity of prime numbers. It is a quotient of the division of (the even numbers starting from the number 4, provided that It has four factors) on the number 2, and the validity of this hypothesis was confirmed according to a regular and proven equation I arrived at another conclusion, which is that the rest of the even numbers that are equal to the number 8 and greater than it and that have factors of 6 or more produce complex numbers for us, meaning that here we also obtained the distribution of the complex numbers in ascending order, starting from the number 4 and so on to infinity of complex numbers. The order of these numbers is actually regular and according to a regular and proven equation.

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